

Ice Sheet System model

Mesh Generation

Chris BORSTAD¹, Bao DUONG^{5,1}, Feras HABBAL^{2,1}, Daria HALKIDES^{1,3},
Michiel HELSEN², Eric LAROUR¹, **Mathieu MORLIGHEM**², Lan NGUYEN^{5,1},
Gilberto PÉREZ^{4,1}, Eric RIGNOT^{2,1}, John SCHIERMEIER¹,
Nicole SCHLEGEL¹, Hélène SEROUSSI¹

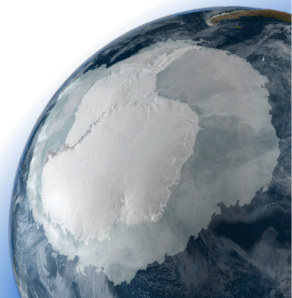
¹Jet Propulsion Laboratory - California Institute of Technology

²University of California, Irvine

³Joint Institute for Regional Earth System Science & Engineering, UCLA

⁴University of Southern California

⁵Cal Poly Pomona



Mesh generation

Outline

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

1 Introduction

2 Squaremesh

Usage

Example

3 Roundmesh

Usage

Example

4 Triangle

Usage

Example

5 Bamg

Uniform mesh

Non-uniform mesh

6 Mesh adaptation

Mesh refinement strategy

Hands-on example

Mesh generation

Mesh generation in ISSM

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

- Mesh generation is crucial for ice sheet modeling

→ controls the space of solutions

- Finer mesh more precise but more computationally intensive
- ISSM has 4 main meshers:
 - ① squar mesh (for ISMIP tests)
 - ② round mesh (for EISMINT tests)
 - ③ triangle (from J. Shewchuk)
 - ④ bamg (adapted from F. Hecht)

Mesh generation

Usage

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

`squaremesh` generates structured uniform meshes for rectangular domain
→ needed for ISMIP tests

```
1 md=squaremesh(model,100,200,15,25);
```

Arguments:

- 1 model
- 2 x-length
- 3 y-length
- 4 number of nodes along the x axis
- 5 number of nodes along the y axis

Mesh generation

Example

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

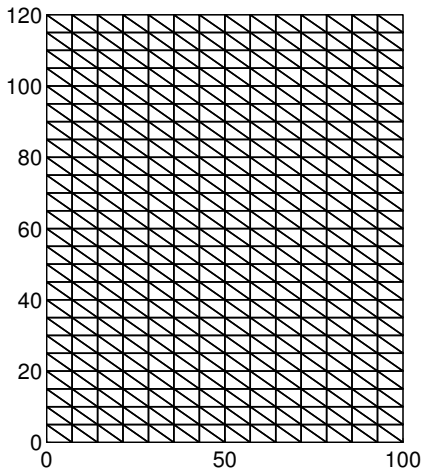
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

```
1 md=squaremesh(model,100,200,15,25);
```



Mesh generation

Usage

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

`roundmess` generates unstructured uniform meshes for circular domain
→ needed for EISMINT tests

```
1 md=roundmesh(model,100,10);
```

Arguments:

- 1 model
- 2 radius
- 3 element size

Mesh generation

Example

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

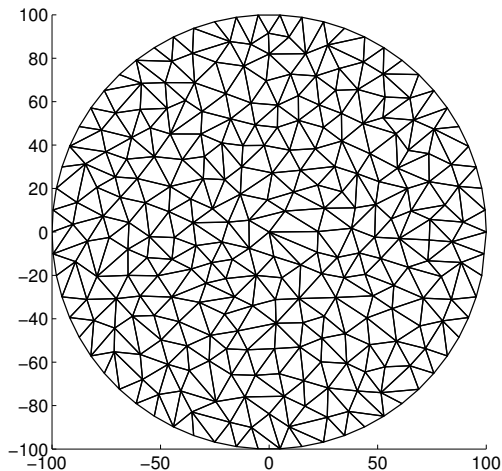
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

```
1 md=roundmesh(model,100,10);
```



Mesh generation

Usage

triangle is a very fast algorithm for mesh generation

- + excellent for uniform mesh
- bad at mesh refinement

```
1 md=triangle(model, 'Square.exp', .2);
```

Arguments:

- 1 model
- 2 ARGUS file of the domain outline

```
## Name:domainoutline
## Icon:0
# Points Count Value
5 1.
# X pos Y pos
0 0
1 0
1 1
0 1
0 0
```

- 3 average element size

Mesh generation

Example

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

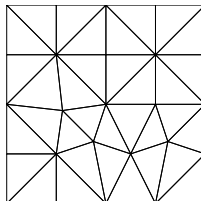
Non-uniform mesh

Mesh adaptation

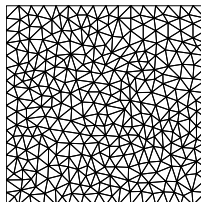
Mesh refinement
strategy

Hands-on example

```
1 md=triangle(model, 'Square.exp', .2);
```



```
1 md=triangle(model, 'Square.exp', .05);
```



History

Initial software:

- BAMG: Bidimensional Anisotropic Mesh Generator
- developed by Frédéric Hecht (INRIA/université de Jussieu)
- released in 2006 after more than 10 years of development
- now part of FreeFEM++

In ISSM:

- entirely rewritten
- usual ISSM interface

Advantages:

- + anisotropic mesh adaptation capability
- not good for uniform meshes

Mesh generation

Usage

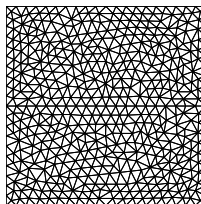
Arguments:

- 1 model
- 2 pair of options (see help)

To create a uniform mesh:

- 1 'domain' followed by the domain name
- 2 'hmax' followed by the triangle size

```
1 md=bamg (model, 'domain', 'Square.exp', 'hmax', .05);
```



- Not as randomly distributed as `triangle`

Mesh generation

Usage

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

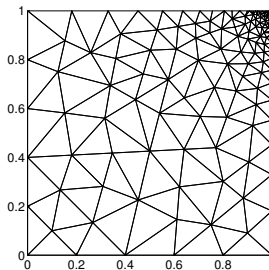
Mesh refinement
strategy

Hands-on example

To create a non-uniform mesh:

- 1 'domain' followed by the domain name
- 2 'hvertices' followed by the element size for each vertex of the domain outline

```
1 hvertices=[0.2;0.2;0.005;0.2];  
2 md=bamg(model,'domain','Square.exp','hVertices',hvertices);
```



Mesh generation

Finite element method

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

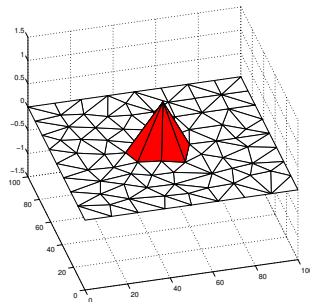
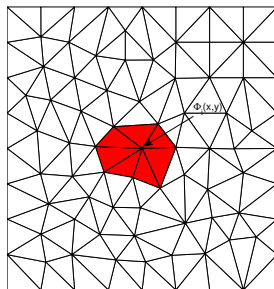
Mesh adaptation

Mesh refinement

strategy

Hands-on example

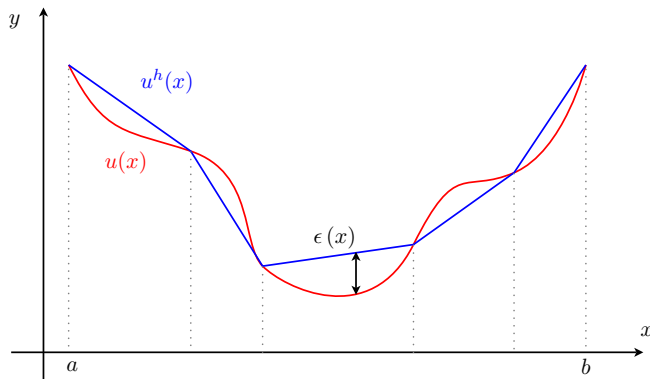
$$v = \sum_{i=1}^N v_i \Phi_i(x, y) \quad (1)$$



Mesh generation

Interpolation error

- We generally use piecewise linear elements ($P1$)
- How to minimize interpolation error and the number of elements at the same time?



Mesh generation

Constant field

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

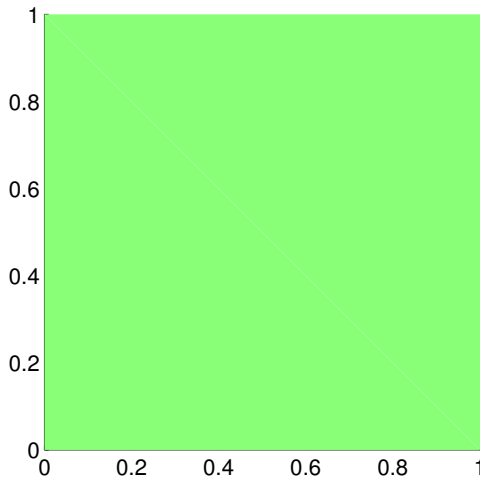
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

Constant field



Mesh generation

Costant field

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

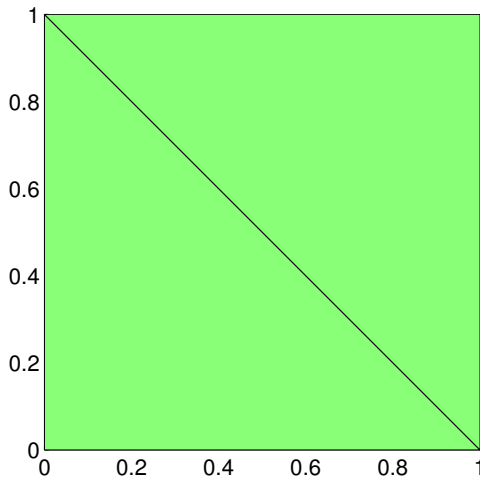
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

Constant field



- Coarse elements OK for constant field

Mesh generation

Linear field

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

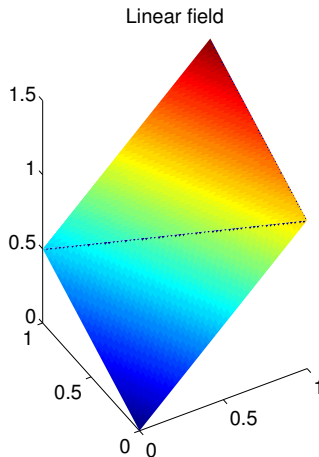
Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example



Mesh generation

Linear field

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

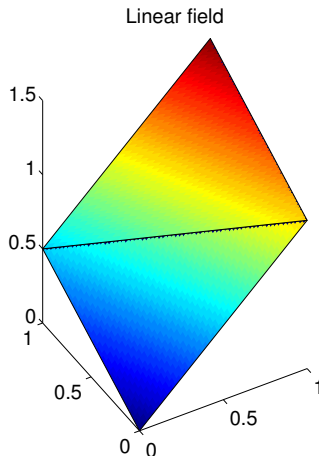
Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example



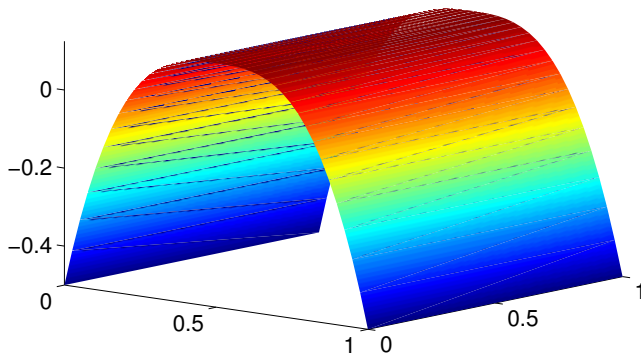
- Coarse elements OK for linear field

[Mesh generation](#)

Non-Linear field

[Introduction](#)[Squaremesh](#)[Usage](#)[Example](#)[Roundmesh](#)[Usage](#)[Example](#)[Triangle](#)[Usage](#)[Example](#)[Bamg](#)[Uniform mesh](#)[Non-uniform mesh](#)[Mesh adaptation](#)[Mesh refinement
strategy](#)[Hands-on example](#)

Non linear field



Mesh generation

Non-Linear field

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

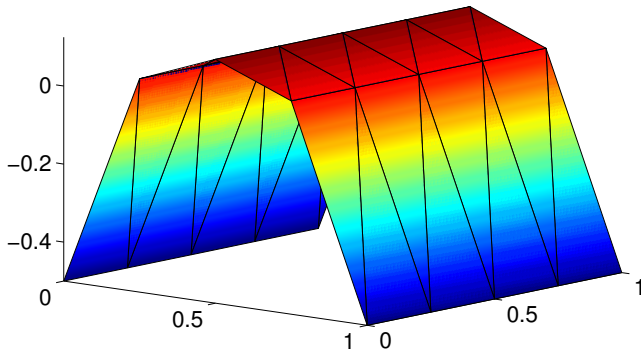
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

25 elements



Mesh generation

Non-Linear field

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

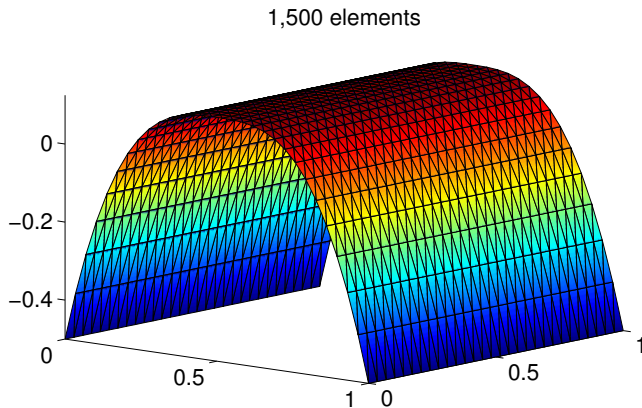
Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example



Mesh generation

Non-Linear field

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

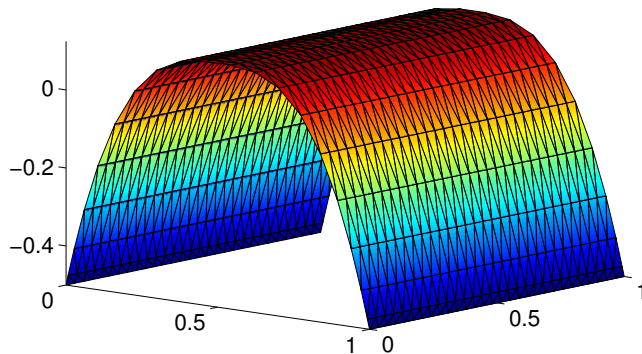
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

1,000 elements



Mesh generation

Non-Linear field

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

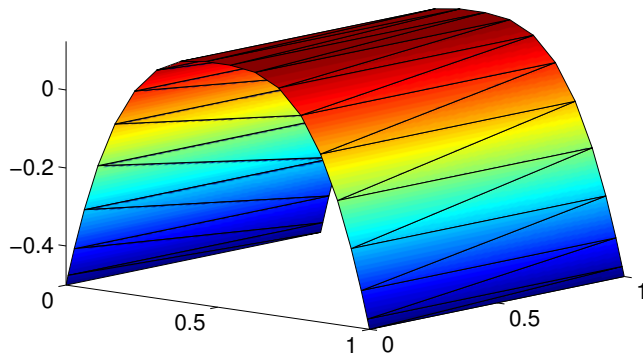
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

40 elements



Mesh generation

Anisotropic mesh refinement

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

Strategy:

- Minimize the interpolation error for a given field
- Metric based on field's Hessian matrix (second derivative)

Mesh generation

Anisotropic mesh refinement

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

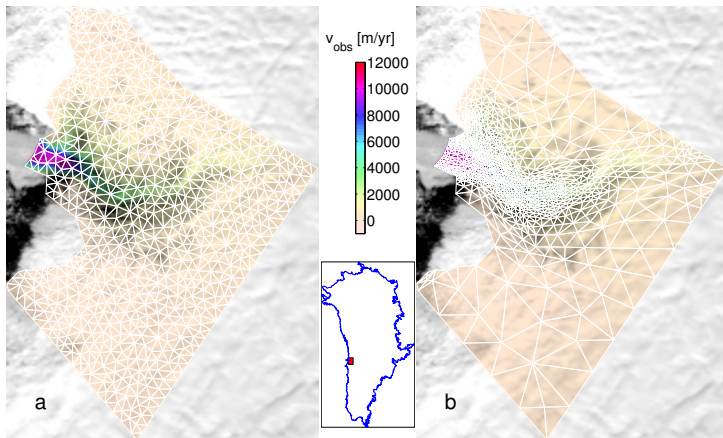
Mesh adaptation

Mesh refinement
strategy

Hands-on example

Strategy:

- Minimize the interpolation error for a given field
- Metric based on field's Hessian matrix (second derivative)



Mesh generation

Field to capture

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

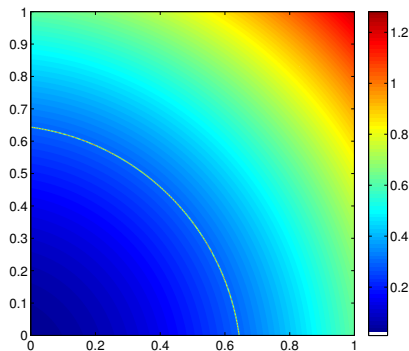
Mesh adaptation

Mesh refinement
strategy

Hands-on example

$$f(x, y) = \exp\left(-\left(\frac{r - 0.75}{\varepsilon}\right)^2\right) + 0.5r^2$$

with $\varepsilon = 0.25$ and $r = (x + 0.1)^2 + (y + 0.1)^2$



Mesh generation

Uniform mesh

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

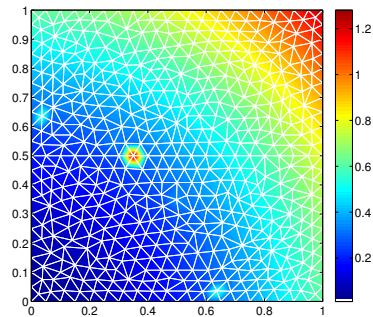
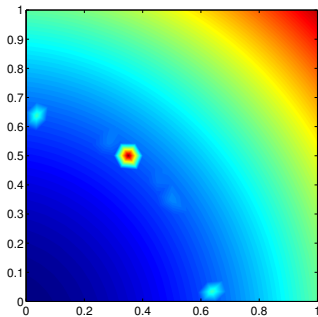
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

```
1 md=bamg(model,'domain','Square.exp','hmax',.05);  
2 vel=shock(md.mesh.x,md.mesh.y);  
3 plotmodel(md,'data',vel,'edgecolor','w');
```



Mesh generation

Mesh refinement

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

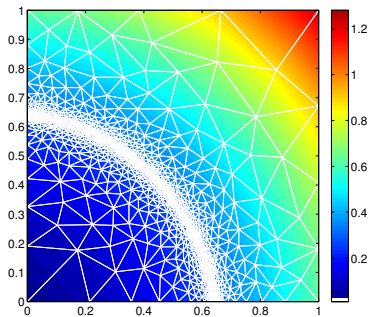
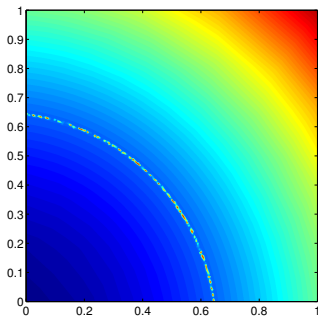
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

```
1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .005);
2 vel=shock(md.mesh.x,md.mesh.y);
3 md=bamg(md, 'field', vel, 'err', 0.05, 'hmin', 0.005, 'hmax', 0.3);
4 vel=shock(md.mesh.x,md.mesh.y);
5 plotmodel(md, 'data', vel, 'edgecolor', 'w');
```



Mesh generation

Mesh refinement

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

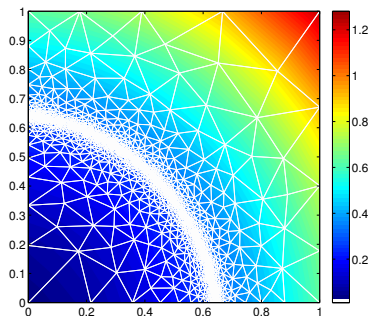
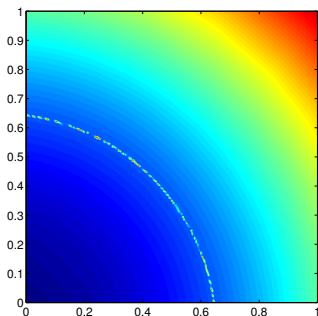
Mesh adaptation

Mesh refinement

strategy

Hands-on example

```
1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .005);  
2 vel=shock(md.mesh.x,md.mesh.y);  
3 md=bamg(md, 'field', vel, 'err', 0.03, 'hmin', 0.005, 'hmax', 0.3);  
4 vel=shock(md.mesh.x,md.mesh.y);  
5 plotmodel(md, 'data', vel, 'edgecolor', 'w');
```



Mesh generation

Mesh refinement

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

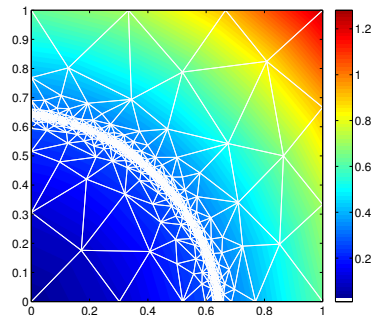
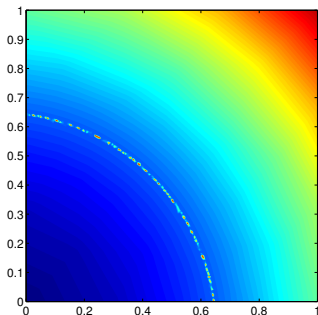
Mesh refinement
strategy

Hands-on example

```

1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .005);
2 vel=shock(md.mesh.x,md.mesh.y);
3 md=bamg(md, 'field', vel, 'err', 0.03, 'hmin', 0.005, 'hmax', 0.3, 'gradation', 3);
4 vel=shock(md.mesh.x,md.mesh.y);
5 plotmodel(md, 'data', vel, 'edgecolor', 'w');

```



Mesh generation

Mesh refinement

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

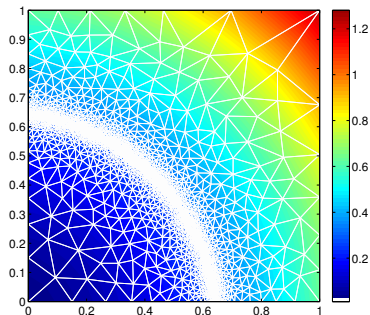
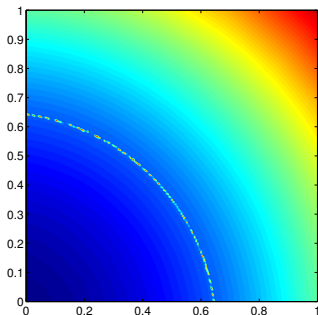
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

```
1 md=bamg(model,'domain','Square.exp','hmax',.005);
2 vel=shock(md.mesh.x,md.mesh.y);
3 md=bamg(md,'field',vel,'err',0.03,'hmin',0.005,'hmax',0.3,'gradation',1.3,'anisomax',1);
4 vel=shock(md.mesh.x,md.mesh.y);
5 plotmodel(md,'data',vel,'edgecolor','w');
```



Mesh generation

Mesh refinement

Introduction

Squaremesh

Usage

Example

Roundmesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

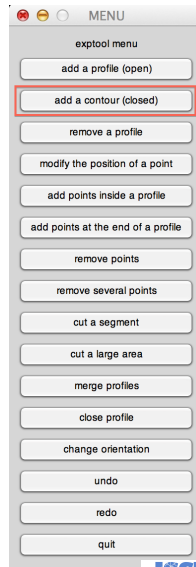
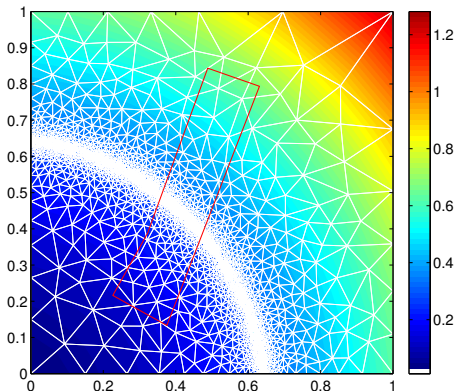
Non-uniform mesh

Mesh adaptation

Mesh refinement
strategy

Hands-on example

```
1 plotmodel(md, 'data', vel, 'edgecolor', 'w');  
2 exptool('refinement.exp')
```



Mesh generation

Mesh refinement

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

Mesh refinement

strategy

Hands-on example

```

1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .005);
2
3 h = NaN*ones(md.mesh.numberofvertices,1);
4 in = ContourToNodes(md.mesh.x,md.mesh.y, 'refinement.exp',1);
5 h(find(in))=0.02;
6 plotmodel(md, 'data', in);
7
8 vel=shock(md.mesh.x,md.mesh.y);

```

```

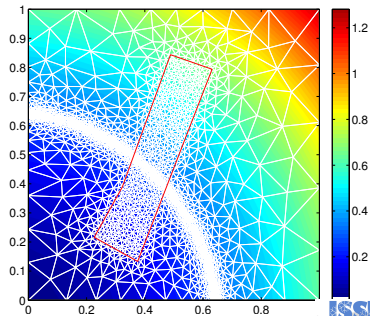
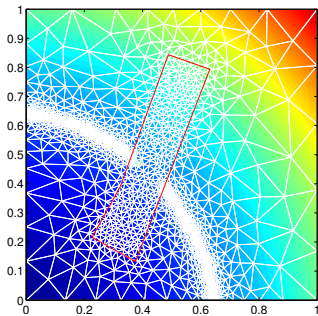
1 md=bamg(md, 'field', vel, 'err', 0.03, 'hmin', 0.005, 'hmax', 0.3, 'hVertices', h);

```

```

1 md=bamg(md, 'field', vel, 'err', 0.03, 'hmin', 0.005, 'hmax', 0.3, 'hmaxVertices', h);

```



Mesh generation

Example 2

Introduction

Squar mesh

Usage

Example

Round mesh

Usage

Example

Triangle

Usage

Example

Bamg

Uniform mesh

Non-uniform mesh

Mesh adaptation

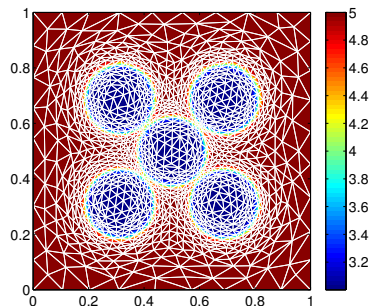
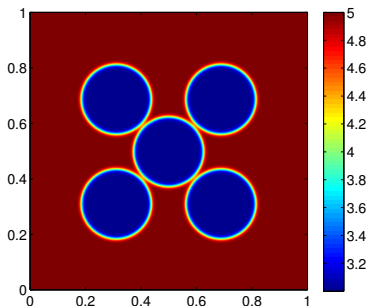
Mesh refinement
strategy

Hands-on example

Similar example with `circles.m`

$$f(x, y) = \tanh\left(30\left(u^2 + v^2 - \varepsilon\right)\right) + \tanh\left(30\left((u - 0.75)^2 + (v - 0.75)^2 - \varepsilon\right)\right) + \tanh\left(30\left((u - 0.75)^2 + (v - 0.75)^2 + \varepsilon\right)\right) + \tanh\left(30\left((u - 0.75)^2 - (v - 0.75)^2 + \varepsilon\right)\right) + \tanh\left(30\left((u - 0.75)^2 - (v - 0.75)^2 - \varepsilon\right)\right)$$

$$\text{with } \varepsilon = 0.25 \quad \text{and} \quad u = 4x - 2, \quad v = 4y - 2$$



Thanks!

