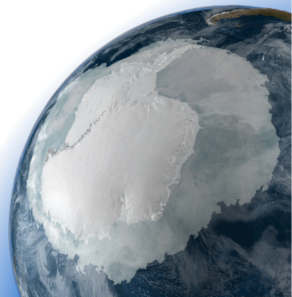


Mesh Generation

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Mesh generation

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Mesh generation in ISSM

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- Mesh generation is crucial for ice sheet modeling

→ controls the space of solutions

- Finer mesh more precise but more computationally intensive
- ISSM has 3 main meshers:
 - ① squaremesh (for ISMIP tests)
 - ② triangle (from J. Shewchuk)
 - ③ bamg (adapted from F. Hecht)

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Usage

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`squaremesh` generates structured uniform meshes for rectangular domain

→ needed for ISMIP tests

```
1 md=squaremesh(model,100,200,15,25);
```

Arguments:

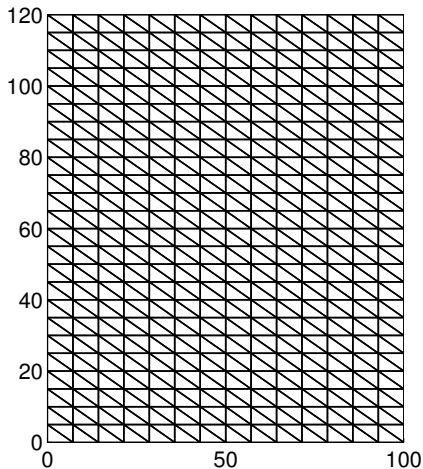
- 1 model
- 2 x-length
- 3 y-length
- 4 number of nodes along the x axis
- 5 number of nodes along the y axis

Mesh generation

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Example

```
1 md=squaremesh(model,100,200,15,25);
```



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Usage

triangle is a very fast algorithm for mesh generation

- + excellent for uniform mesh
- bad a mesh refinement

```
1 md=triangle(model, 'Square.exp', .2);
```

Arguments:

- 1 model
- 2 ARGUS file of the domain outline

```
## Name:domainoutline
## Icon:0
# Points Count Value
5 1.
# X pos Y pos
0 0
1 0
1 1
0 1
0 0
```

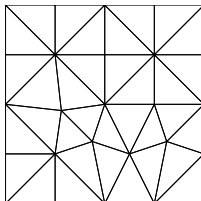
- 3 average element size

Mesh generation

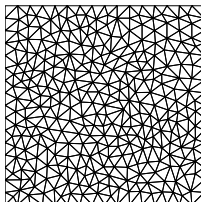
Larour et al.

Example

```
1 md=triangle(model, 'Square.exp', .2);
```



```
1 md=triangle(model, 'Square.exp', .05);
```



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Initial software:

- BAMG: Bidimensional Anisotropic Mesh Generator
- developed by Frédéric Hecht (INRIA/université de Jussieux)
- released in 2006 after more than 10 years of development
- now part of FreeFEM++

In ISSM:

- almost entirely rewritten
- usual ISSM interface

Advantages:

- + anisotropic mesh adaptation capability
- not good for uniform meshes

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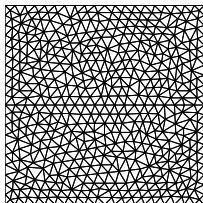
Arguments:

- 1 model
- 2 pair of options (see help)

To create a uniform mesh:

- 1 'domain' followed by the domain name
- 2 'hmax' followed by the triangle size

```
1 md=bamg (model, 'domain', 'Square.exp', 'hmax', .05);
```



- Not as randomly distributed as `triangle`

Mesh generation

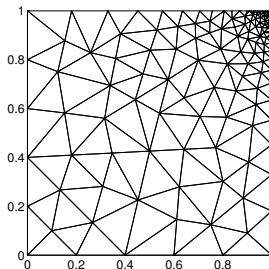
Larour et al.

Usage

To create a non-uniform mesh:

- 1 'domain' followed by the domain name
- 2 'hvertices' followed by the element size for each vertex of the domain outline

```
1 hvertices=[0.2;0.2;0.005;0.2];  
2 md=bamg(model,'domain','Square.exp','hVertices',hvertices);
```

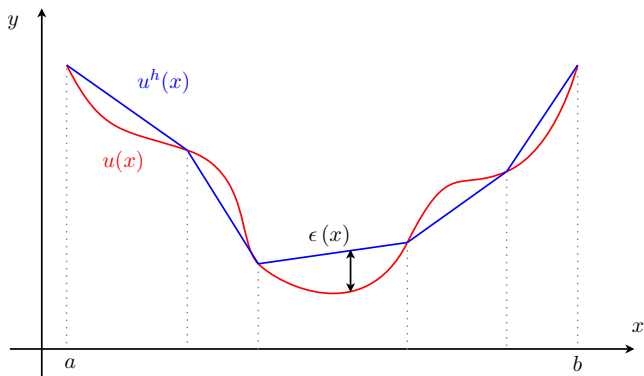


Mesh generation

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Interpolation error

- We generally use piecewise linear elements ($P1$)
- How to minimize interpolation error and the number of elements at the same time?



Mesh generation

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Constant field

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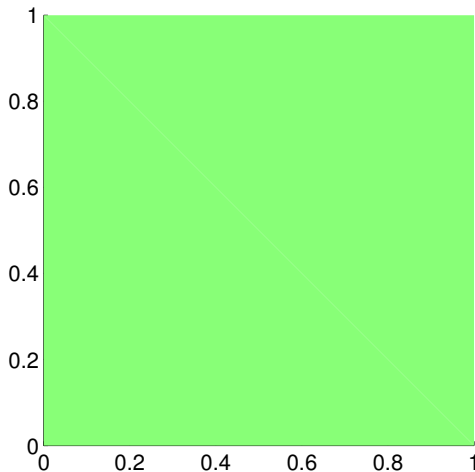
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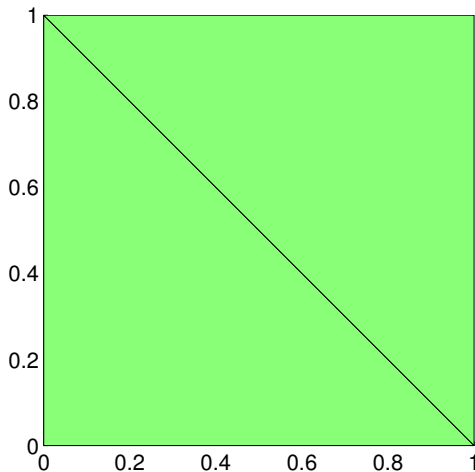
Uniform mesh
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Constant field



- Coarse elements OK for constant field

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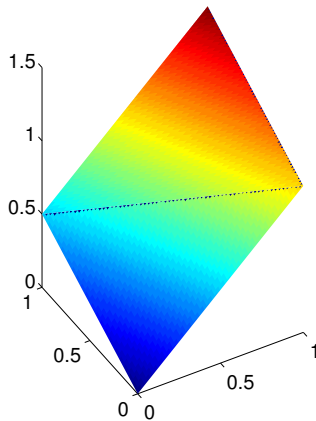
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Linear field



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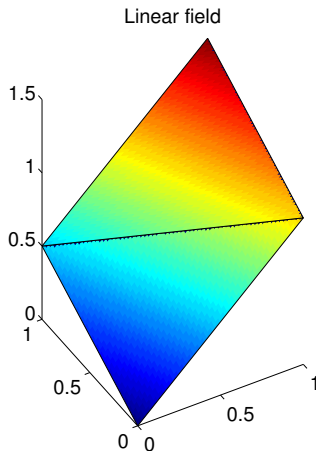
Uniform mesh

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- Coarse elements OK for linear field

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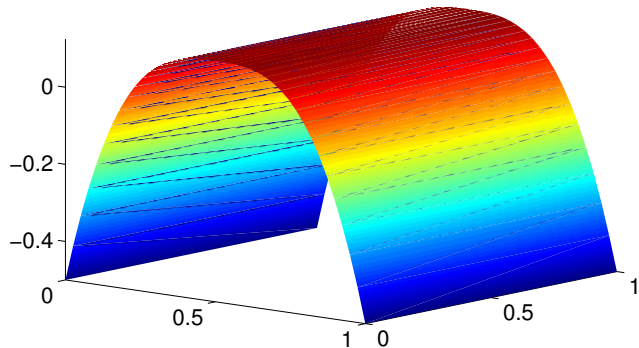
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Non linear field



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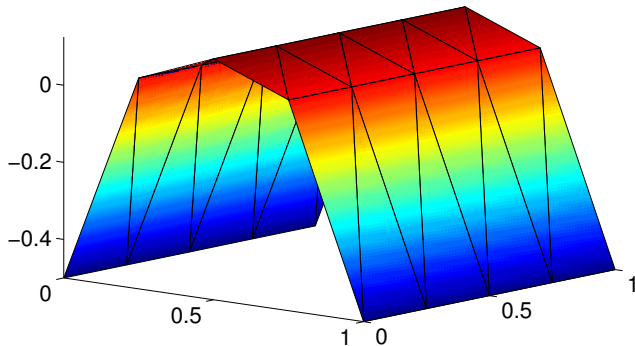
Non-uniform mesh

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25 elements



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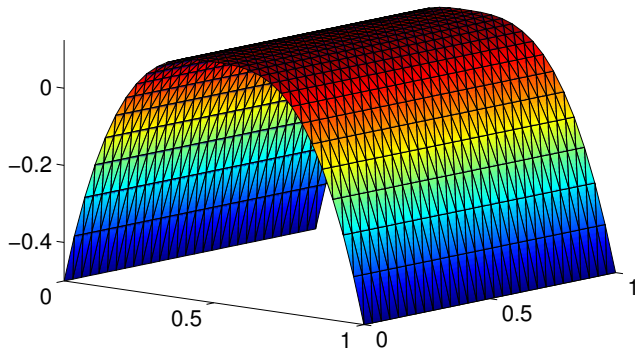
Non-uniform mesh

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1,500 elements



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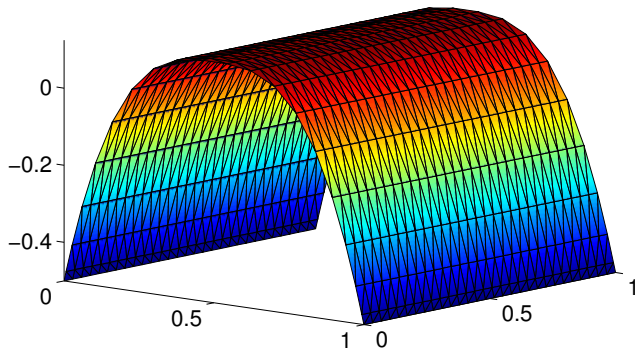
Non-uniform mesh

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1,000 elements



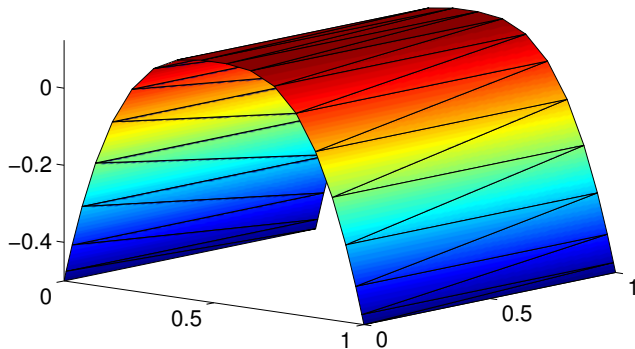
[Mesh generation](#)

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Non-Linear field

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40 elements



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Anisotropic mesh refinement

Strategy:

- Minimize the interpolation error for a given field
- Metric based on field's Hessian matrix (second derivative)

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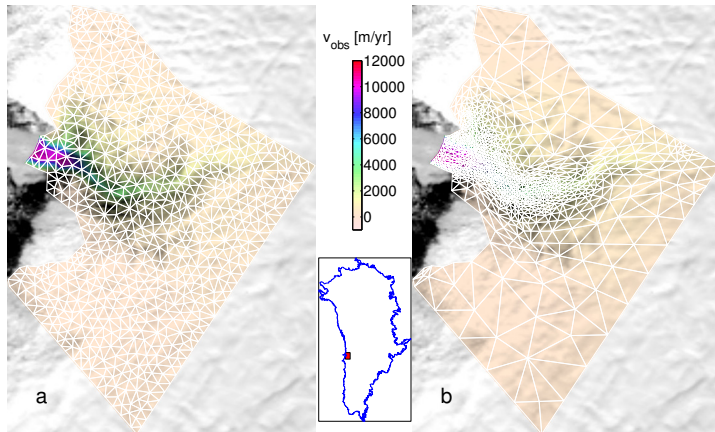
Mesh generation

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Anisotropic mesh refinement

Strategy:

- Minimize the interpolation error for a given field
- Metric based on field's Hessian matrix (second derivative)

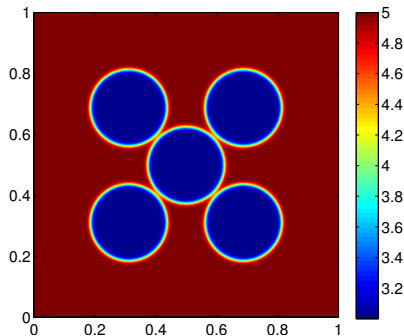


Mesh generation

Larour et al.

Field to capture

$$\begin{aligned} f(x, y) &= \tanh \left(30 \left(u^2 + v^2 - \varepsilon \right) \right) \\ &+ \tanh \left(30 \left((u - 0.75)^2 + (v - 0.75)^2 - \varepsilon \right) \right) + \tanh \left(30 \left((u - 0.75)^2 + (v - 0.75)^2 + \varepsilon \right) \right) \\ &+ \tanh \left(30 \left((u - 0.75)^2 - (v - 0.75)^2 + \varepsilon \right) \right) + \tanh \left(30 \left((u - 0.75)^2 - (v - 0.75)^2 - \varepsilon \right) \right) \\ &\text{with } \varepsilon = 0.25 \quad \text{and} \quad u = 4x - 2, \quad v = 4y - 2 \end{aligned}$$



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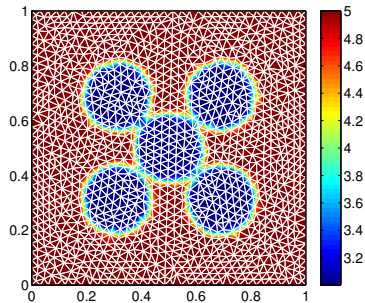
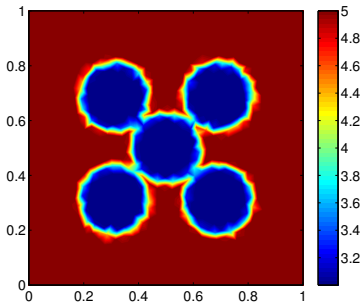
Non-uniform mesh

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Hands-on example

```
1 md=bamg(model,'domain','Square.exp','hmax',.03);  
2 vel=circles(md.mesh.x,md.mesh.y);  
3 plotmodel(md,'data',vel,'edgecolor','w');
```



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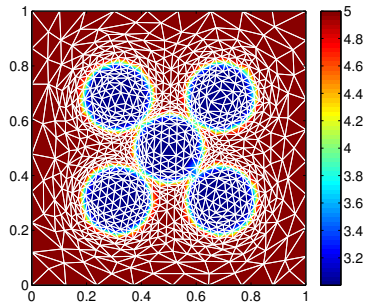
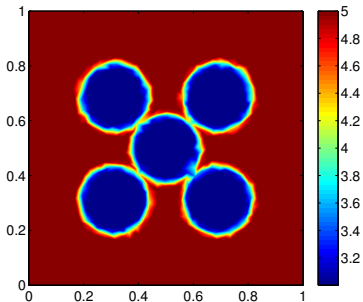
Non-uniform mesh

Mesh adaptation

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```
1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .03);
2 vel=circles(md.mesh.x,md.mesh.y);
3 md=bamg(md, 'field', vel, 'err', 0.05, 'hmin', 0.005, 'hmax', 0.3);
4 vel=circles(md.mesh.x,md.mesh.y);
5 plotmodel(md, 'data', vel, 'edgecolor', 'w');
```



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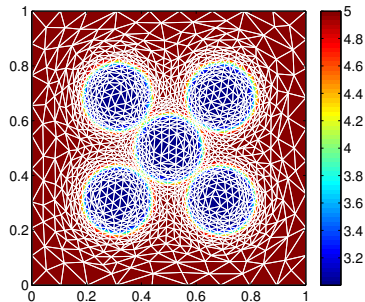
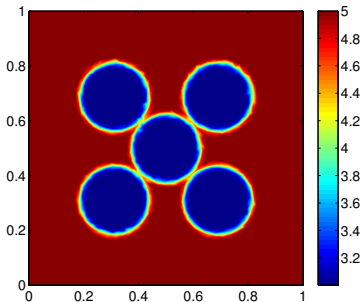
Non-uniform mesh

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```
1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .03);
2 vel=circles(md.mesh.x,md.mesh.y);
3 md=bamg(md, 'field', vel, 'err', 0.03, 'hmin', 0.005, 'hmax', 0.3);
4 vel=circles(md.mesh.x,md.mesh.y);
5 plotmodel(md, 'data', vel, 'edgecolor', 'w');
```



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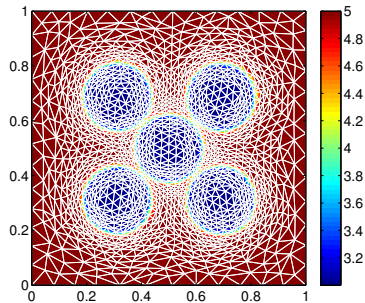
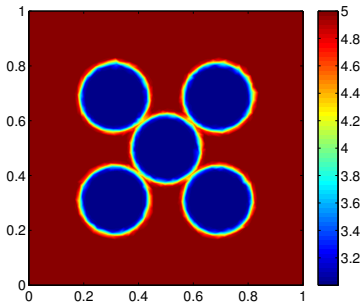
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```
1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .03);  
2 vel=circles(md.mesh.x,md.mesh.y);  
3 md=bamg(md, 'field', vel, 'err', 0.03, 'hmin', 0.005, 'hmax', 0.3, 'gradation', 1.3);  
4 vel=circles(md.mesh.x,md.mesh.y);  
5 plotmodel(md, 'data', vel, 'edgecolor', 'w');
```



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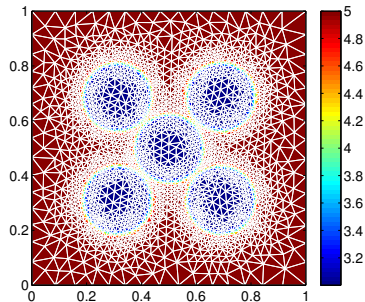
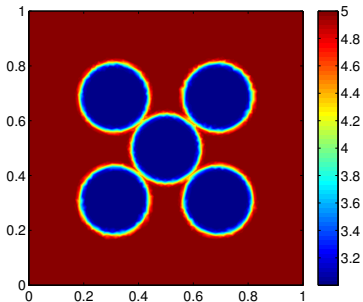
Non-uniform mesh

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```
1 md=bamg(model, 'domain', 'Square.exp', 'hmax', .03);  
2 vel=circles(md.mesh.x,md.mesh.y);  
3 md=bamg(md, 'field', vel, 'err', 0.03, 'hmin', 0.005, 'hmax', 0.3, 'gradation', 1.3, 'anisomax', 1);  
4 vel=circles(md.mesh.x,md.mesh.y);  
5 plotmodel(md, 'data', vel, 'edgecolor', 'w');
```



Thanks!

