

Programmers's manual

Bořek Patzák

Czech Technical University
Faculty of Civil Engineering
Department of Structural Mechanics
Thákurova 7, 166 29 Prague, Czech Republic

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1 Introduction

The aim of OOFEM project is to develop efficient and robust tool for FEM computations as well as to provide modular and extensible environment for future development.

The aim of this document is to provide the introduction to the OOFEM principles and internal structure. However, it is not intended to contain the detailed description of the whole package. Interested readers should follow the OOFEM library reference manual¹ for more detailed reference. This hypertext reference manual is generated automatically from the source code and it represents always up to date source of information. It contains a class hierarchy browser, an alphabetical list of all components and a detailed hypertext description of all library components, including all classes and their attributes and services.

The OOFEM comes divided into two basic parts - a library and a distribution. The library (called OOFEMlib) is the core part of the OOFEM, containing the basic collection of classes. These classes are problem independent and they provide the common definitions and support to all FEM problems. The distribution part contains the rest - modules representing the problem dependent parts.

The OOFEM package is written in C++ (see, for example [11], for reference). This document contains many examples and listings of a parts of the source code. Therefore, the ideal reader should be familiar with C++ programming language. However, any reader with an object-oriented background should follow this document, since the examples are written in a form, which is easy to read and understand.

2 General Structure

General structure is shown in Fig. 1. The key class is **EngineeringModel** representing the problem under consideration. It can contain one or more domains (abstractions for computational mesh) represented by the **Domain** class. Generally speaking, it contains the domain description, or if the program runs in parallel, then it contains the description of the domain associated with a particular processor or thread of execution. **Domain** contains and manages lists of degree of freedom managers, elements, boundary conditions, cross sections, and material models - these describe the geometry of the problem, its constitutive properties, and applied boundary conditions. Services for accessing each of these objects are provided. **Domain** class & object provides services for reading input files and instantiating corresponding components accordingly.

Engng model and Numerical method interfaces, shown schematically in top-left frame, will be explained in Section 3. The classes & objects in the left-bottom frame represent the element, material model, and cross section abstractions. Because of their principal importance, a special section 4 will be devoted to a detailed explanation of this frame.

DOF is an abstraction for a single degree of freedom (DOF). It maintains its physical meaning and associated equation number. **DOF** is the attribute of **DofManager**. The **DofManager** manages the collection of DOFs. A typical derived class is class **Node**, representing a node in a finite element mesh. **Boundary condition** and **Initial condition** are abstractions of boundary and initial conditions. They are attributes of **Domain** and are associated with one or more **DOFs**. The abstract class **Load**, derived from base **Boundary condition** class, is an abstraction for load. It is an attribute of **Domain** and can be associated with several dof managers or elements, according to the type of loading it represents. The class declares the basic services provided by all derived classes. Derived classes declare specific load type dependent services and implement all necessary services.

3 Engineering model — Numerical Method Frame

3.1 Problem representation - Engineering model

Engng model is an abstraction for the problem under consideration. It represents the type of analysis to be performed. Base class declares and implements the basic general services for assembling characteristic components and services

¹<http://ksm.fsv.cvut.cz/oofem/oofem.html>

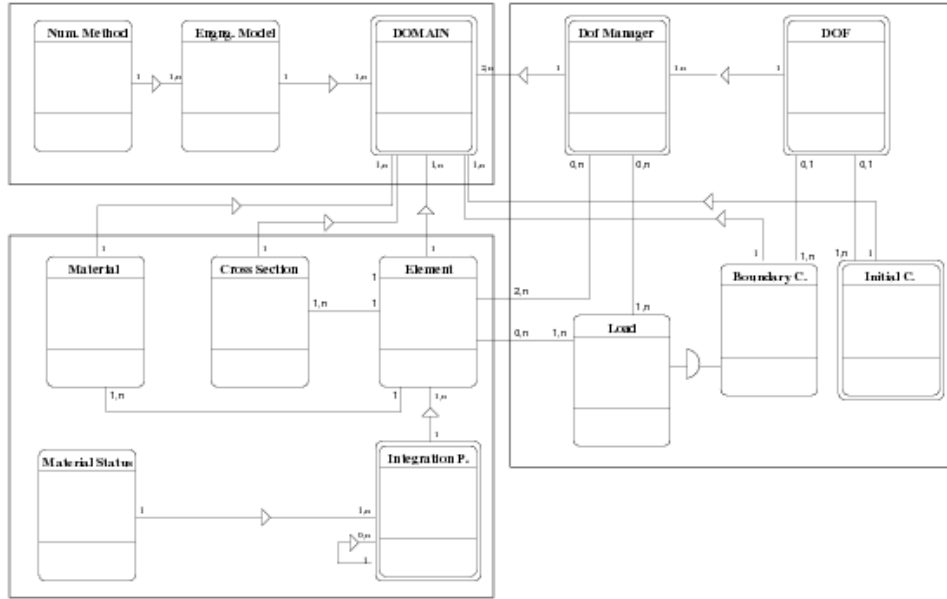


Figure 1: General Structure.

for starting the solution step and its termination. Derived classes “know” the form of governing equation and the physical meaning of particular components. They are responsible for forming the governing equation for each solution step, usually by summing contributions from particular elements and nodes.

The solution step may represent either a time step, a load increment, or a load case. The solution steps are grouped together into so called meta steps. The meta step can be thought as a sequence of solution steps, with the same set of attributes used to drive the behavior of engng model. For each meta step, the engng model typically updates certain attribute values according to meta step model attributes (see `updateAttributes` service) and creates the solution steps accordingly. This allows to switch to a different time increment, a different solution control, etc. If no meta step is specified, the engng model creates a default one for all required solution steps. There are two services, where engng model attributes are set or updated. The first one, used for those attributes which do not vary during the solution of the problem, are initialized in `instantiateYourself` service. The second service is `updateAttributes`, where the attributes (with meta step validity) are updated according to a values valid for the given meta step. If no meta step is introduced, a default one is created (with the attributes set to the engng model attributes). Then there is no difference, whether the attributes are initialized in `instantiateYourself` or in `updateAttributes`, but the preferred scheme is to read all attributes in `instantiateYourself` and to left `updateAttributes` service empty.

The basic EngngModel tasks are following:

- Assembling governing equations by summing contributions (typically from nodes and elements).
- Solving the problem described by governing equation(s) using the instance of a suitable numerical method. This requires to establish a mapping between numerical method-parameters and EngngModel components of governing equation. EngngModel must map each component of governing equation(s) (which has physical meaning) to the corresponding numerical component. This mapping between physical components to independent numerical components (understood by the numerical method) is important, because it allows the numerical method to be used by many EngngModels with different component meaning and allows to use different numerical methods by EngngModel. This is achieved by using the compulsory numerical component names (see further).
- Providing access to the problem solution. Services for returning unknown values according to their type and mode are provided. These services are used by DOFs to access their corresponding unknowns.

- Terminating the time step by updating the state of the problem domains (nodes and elements, including integration points).
- Managing the problem domains, meta steps, and problem input/output streams.
- Equation numbering.
- Storing and restoring problem state to/from context file.
- Updating the DOF unknown dictionaries if EngngModel supports changes of static system (see the Dof class documentation for detailed explanation). In general, if the static system changes are not supported, then when the DOFs are asked for unknowns, they use their equation numbers to ask EngngModel for corresponding unknowns. The unknown values are therefore stored in the EngngModel and are requested by the DOFs. On the other hand, when the changes of the static system are supported, the equation numbers of DOFs can vary during the solution. Therefore, so called DOF unknown dictionaries are introduced. All unknowns are stored on the DOF level and the DOFs will use in such case their own dictionaries instead of requesting the EngngModel. The EngngModel is then fully responsible for updating this dictionaries for each DOF with all necessary unknowns (see updateDofUnknownsDictionary function).

The complete listing of **EngngModel** class declaration follows.

```
/* $Header: /home/cvs/bp/oofem/doc/usrman/usrman.tex,v 1.7 2003/04/23 14:19:27 bp Exp $ */

// *****
// *** CLASS ENGNGMODEL ***
// *****
class EngngModel
{

protected:
    /// number of receiver domains
    int ndomains;
    /// List of problem domains
    AList<Domain>* domainList;
    /// Total number of time steps
    int numberOfSteps;
    /// total number of equation in cuurent time step
    int numberOfEquations;
    /// total number or prescribed equations in current time step
    int numberOfPrescribedEquations;
    /// number of equations per domain
    IntArray domainNeqs;
    /// number of prescribed equations per domain
    IntArray domainPrescribedNeqs;
    /// renumbering flag
    int renumberFlag;
    /// equation numbering completed flag
    int equationNumberingCompleted;
    /// number of meta steps
    int nMetaSteps;
    /// List of problem metasteps
    AList<MetaStep>* metaStepList;
    /// Solution step when IC (initial conditions) apply
    TimeStep* stepWhenIcApply;
```

```

/// Currnet time step
TimeStep* currentStep ;
/// Previous time step
TimeStep* previousStep;
/// receivers id
int      number ;

/// Path to input stream
//char*      dataInputFileName ;
/// Path to output stream
char*      dataOutputFileName ;
/// Output stream
FILE* outputStream;
/// Input stream
//FILE* inputStream;
/// Domain context output mode
ContextOutputMode      contextOutputMode;
int      contextOutputStep;

///Export module manager
ExportModuleManager* exportModuleManager;

/// Domain mode
problemMode      pMode ;
/// solution start time
time_t startTime;
// initial value of processor time used by program
// clock_t startClock;

/// master e-model; if defined receiver is in maintained (slave) mode
EngngModel* master;
/// context
EngngModelContext* context;
/**
    flag indicating that the receiver runs in parallel.
*/
int parallelFlag;

public:
/**
    Constructor. Creates Engng model with number i belonging to domain d.
*/
EngngModel (int i, EngngModel* _master = NULL) ;    // constructor
/**
    Constructor. Creates Engng model with number i and input file given by path.
*/
EngngModel (int i, char* s, EngngModel* _master = NULL);
/// Destructor.
virtual ~EngngModel () ;    // destructor

/**
    Service for accessing particular problem domain.

```

```

Generates error if no such domain is defined.
@param: n pointer to n-th domain is returned
*/
Domain*      giveDomain (int n);
/// Returns number of domains in problem.
int          giveNumberOfDomains () {return ndomains;}
/** Service for accessing ErrorEstimator corresponding to particular domain */
virtual ErrorEstimator* giveDomainErrorEstimator (int n) {return NULL;}
// input / output
/// Returns input file path.
//char*      giveInputDataFileName () ;
/// Returns file descriptor of output file
FILE*       giveOutputStream () ;
/** Returns base output file name
    to which extensions, like .out .vtk .osf should be added.
    In current implementation, output file name is simply returned.
    @param path and base file name will be copied into the array pointed to by dest
    @param not more than n bytes of src are copied
*/
char * giveOutputBaseFileName(char *dest, size_t n)
    {return strncpy (dest, dataOutputFileName, n);}

//FILE*      giveInputStream () ;

/*
Returns current time in seconds as returned by time call.
@return current time in time_t structure.
*/
//time_t     getTime ();
/*
Returns an approximation of processor time used by the program.
The value returned is the CPU time used so far as a
clock_t; to get the number of seconds used, divide by
CLOCKS_PER_SEC. Calls clock ANSI C function.
The C standard allows for arbitrary values at the start of
the program; take the difference between the value
returned from a call to this method at the start of the pro-
gram and the end to get maximum portability.
*/
//clock_t    getClock ();
/**
Returns domain context output mode.
*/
ContextOutputMode giveContextOutputMode () {return contextOutputMode;}
/**
Returns domain context output step.
*/
int          giveContextOutputStep () {return contextOutputStep;}
/**
Sets context output mode of receiver.
@param contextMode domain context mode.
*/
void         setContextOutputMode (ContextOutputMode contextMode)
    {contextOutputMode=contextMode;}

```



```

/**
 Sets user defined context output mode (it sets contextOutputMode to contextOutputMode),
 setting contextOutputStep to given value.
 @param cStep new context output step
 */
void          setUDContextOutputMode (int cStep)
 {contextOutputMode=USERDEFINED; contextOutputStep = cStep;}
/**
 Sets domain mode to given mode.
 @param mode domain mode.
 */
void          setProblemMode (problemMode mode) {pMode = mode;}
/// Returns domain mode.
problemMode   giveProblemMode ()          {return pMode;}
/// Sets the renumber flag to TRUE
virtual void   setRenumberFlag() {this->renumberFlag = 1;}
/// Sets the renumber flag to FALSE
virtual void   resetRenumberFlag() {this->renumberFlag = 0;}

/**
 Performs analysis termination after finishing analysis.
 */
void          terminateAnalysis () ;

// solving

/** Starts solution process. Implementation should invoke for each
 time step solveYourselfAt function with time step as parameter. Time
 steps are created using giveNextStep function (this will set current
 time step to newly created, and updates previous step).
 */
virtual void   solveYourself ();

/** Solves problem for given time step. Should assemble
 characteristic matrices and vectors if necessary and solve problem
 using appropriate numerical method. After finishing solution,
 this->updateYourself function for updating solution state and then
 this->terminate function (for updating nodal and element values)
 should be called.
 */
virtual void   solveYourselfAt (TimeStep*) {}
//virtual int   requiresNewLhs () {return 1;}

/** Terminates the solution of time step. Default implementation
 calls prinOutput() service and if specified, context of whole domain
 is stored and output for given time step is printed.
 */
virtual void   terminate (TimeStep*);

/** Prints the output of the solution step (using virtual
 this->printOutputAtservice) to the stream determined using
 this->giveOutputStream() method and calls exportModuleManager to do
 output.
 */
virtual void   doStepOutput(TimeStep*);

```

```

/** Saves context of given solution step, if required (determined
using this->giveContextOutputMode() method).
*/
void                                saveStepContext(TimeStep*);

/** Updates internal state after finishing time step. (for example
total values may be updated according to previously solved
increments). Then element values are also updated (together with
related integration points and material statuses). */
virtual void                        updateYourself (TimeStep* stepN);

/** Provides the opportunity to initialize state variables stored in
element integration points according to initial conditions using
function initializeYourself() on element level. Should be called
when current time step is time step when IC will apply (see
EngngModel::giveNumberOfTimeStepWhenIcApply) somewhere from
solveYourselfAt function). Implementation must be provided. Default
implementation is empty. */
virtual void                        initializeYourself (TimeStep*) {}

/** Initializes the newly generated discretization state according to
previous solution. This process should typically include restoring
old solution, instantiating newly generated domain(s) and by mapping
procedure. */
virtual int                         initializeAdaptive (int stepNumber) {return 0;}

/** Returns total number of equations in active (current time step)
time step. The UnknownType parameter allows to distinguish between
several possible governing equations, that can be numbered
separately. */
virtual int                         giveNumberOfEquations (EquationID);

/** Returns total number of prescribed equations in active (current
time step) time step. The UnknownType parameter allows to
distinguish between several possible governing equations, that can be
numbered separately. */
virtual int                         giveNumberOfPrescribedEquations (EquationID);

/** Returns number of equations for given domain in active (current
time step) time step. The UnknownType parameter allows to
distinguish between several possible governing equations, that can be
numbered separately. */
virtual int                         giveNumberOfDomainEquations (int, EquationID);

/** Returns number of prescribed equations for given domain in active
(current time step) time step. The UnknownType parameter allows to
distinguish between several possible governing equations, that can be
numbered separately. */
virtual int                         giveNumberOfPrescribedDomainEquations (int, EquationID);
//virtual IntArray*                  GiveBanWidthVector ();

// management components

```

```

/** Provides backward mapping between numerical component and
characteristic component on EngngModel level. */
virtual CharType giveTypeOfComponent (NumericalCmpn) {return UnknownCharType;}

/** Returns requested unknown. Unknown at give time step is
characterized by its type and mode and by its equation number. This
function is used by Dofs, when they are requested for their associated
unknowns. @see Dof::giveUnknown method */
virtual double giveUnknownComponent (EquationID, ValueModeType, TimeStep*, Domain*, Dof*)
{return 0.0;}
virtual double giveUnknownComponent (UnknownType, ValueModeType, TimeStep*, Domain*, Dof*)
{return 0.0;}

/** Initializes whole problem according to its description stored in
inputStream. Prints header, opens the outFileName, instantiate
itself the receiver using virtual initializeFrom service and
instanciates all problem domains. */
virtual int instantiateYourself (DataReader* dr, InputRecord* ir,
                                char* outFileName, char* desc) ;

/** Initializes receiver according to object description in input
reader. InitString can be imagined as data record in component
database belonging to receiver. Receiver may use value-name
extracting functions to extract particular field from record.*/
virtual IRResultType initializeFrom (InputRecord* ir);
/// Instantiate problem domains by calling their instantiateYourself() service
int instantiateDomains (DataReader* dr);
/// Instantiate problem meta steps by calling their instantiateYourself() service
int instantiateMetaSteps (DataReader* dr);
/// Instantiate default metastep, if nmsteps is zero
int instantiateDefaultMetaStep (InputRecord* ir);

/** Update receiver attributes according to step metaStep
attributes. Allows the certain parameters or attributes to be
updated for particular metastep. The metastep provides the
attributes record, from which the corresponding attributes can be
read. The service takes TimeStep as parameter, from which
corresponding MetaStep is requested. It is recommended, to implement
this service in such way, that multiple calls for steps belonging to
same MetaStep does not change response. The default implementation
updates the numerical method attributes. @param TimeStep time step.
*/
virtual void updateAttributes (TimeStep*);

/** Update e-model attributes according to step metaStep
attributes. Calls updateAttributes. At the end the meta step input
reader finish() service is called in order to allow for unread
attribute check. */
void initMetaStepAttributes (TimeStep* tStep);

/** Stores the state of model to output stream. Stores not only the
receiver state, but also same function is invoked for all DofManagers
and Elements in associated domain. Note that by storing element
context also contexts of all associated integration points (and

```

```

material statuses) are stored. Stored context is associated with
current time step. One time step can have only one associated
context. Multiple call to saveContext within same time step override
previously saved context for this step. By default the stream
parameter is used to store data and is not closed. If stream is
NULL, new file descriptor is created and this must be also closed at
the end.
@param stream - context stream. If NULL then new file
descriptor will be opened and closed at the end else the stream
given as parameter will be used and not closed at the end.
@return contextIOResultType.
@exception throws an ContextIOERR exception if error encountered */
virtual contextIOResultType saveContext (FILE *stream, void *obj = NULL) ;

/** Restores the state of model from output stream. Restores not only
the receiver state, but also same function is invoked for all
DofManagers and Elements in associated domain. Note that by restoring
element context also contexts of all associated integration points
(and material statuses) are restored. Each context is associated
with unique time step. Only one context per time step is
allowed. Restore context function will restore such context, which is
related (through its step number) to time step number and version
given in obj parameter. Restoring context will change current time
step in order to correspond to newly restored context.
@param stream context file
@param obj is a void pointer to an int array containing two values:time step
version of a context file to be restored.
@return contextIOResultType.
@exception throws an ContextIOERR exception if error encountered.
*/
virtual contextIOResultType restoreContext (FILE* stream, void* obj = NULL)

/** Updates domain links after the domains of receiver have
changed. Used mainly after restoring context - the domains may change
and this service is then used to update domain variables in all
components belonging to receiver like errorestimators, solvers, etc,
having domains as attributes. */
virtual void updateDomainLinks() {};
void resolveCorrespondingStepNumber (int&, int&, void* obj);
/// Returns current time step.
TimeStep* giveCurrentStep () {if (master) return master->giveCurrentStep();
                             else return currentStep;}

/// Returns previous time step.
TimeStep* givePreviousStep() {if (master) return master->givePreviousStep();
                             else return previousStep;}

/// Returns next time step (next to current step) of receiver.
virtual TimeStep* giveNextStep() {return NULL;}
/// Returns the solution step when Initial Conditions (IC) apply
virtual TimeStep* giveSolutionStepWhenIcApply()
{if (master) return master->giveCurrentStep();
 else return stepWhenIcApply;}

/// Returns number of first time step used by receiver.
virtual int giveNumberOfFirstStep ()
{if (master) return master->giveNumberOfFirstStep();

```

```

        else return 1;}
    /// Return number of meta steps
    int giveNumberOfMetaSteps () {return nMetaSteps;}
    /// Returns the i-th meta step
    MetaStep* giveMetaStep (int i);
    /// Returns total number of steps.
    int giveNumberOfSteps()
    {if (master) return master->giveNumberOfSteps();
     else return numberOfSteps ;}
    /// Returns end of time interest (time corresponding to end of time integration).
    virtual double giveEndOfTimeOfInterest () {return 0.;}
    /// Returns the time step number, when initial conditions should apply.
    virtual int giveNumberOfTimeStepWhenIcApply()
    {if (master) return master->giveNumberOfTimeStepWhenIcApply();
     else return 0;}
    /// Returns reference to receiver's numerical method
    virtual NumericalMethod* giveNumericalMethod (TimeStep*) {return NULL;}
    /// Returns receiver's export module manager
    ExportModuleManager* giveExportModuleManager() {return exportModuleManager;}

    /** Increases number of equations of receiver's domain and returns
    newly created equation number. Used mainly by DofManagers to
    allocate their corresponding equation number if it is not currently
    allocated. The DofIDItem parameter allows to distinguish between
    several possible governing equations, that can be numbered
    separately. */
    virtual int giveNewEquationNumber (int domain, DofIDItem)
    {return ++domainNeqs.at(domain);}

    /** Increases number of prescribed equations of receiver's domain and
    returns newly created equation number. Used mainly by DofManagers to
    allocate their corresponding equation number if it is not currently
    allocated. The DofIDItem parameter allows to distinguish between
    several possible governing equations, that can be numbered
    separately. */
    virtual int giveNewPrescribedEquationNumber (int domain, DofIDItem)
    {return ++domainPrescribedNeqs.at(domain);}

    /**
    Assigns context file-descriptor for given step number to stream.
    Returns nonzero on success.
    @param stepNumber solution step number to store/restore
    @param stepVersion version of step
    @param cmode determines the i/o mode of context file
    @param errLevel determines the amount of warning messages if
    errors are encountered, level 0 no warnings reported.
    */
    int giveContextFile (FILE** contextFile, int stepNumber, int stepVersion,
    ContextFileMode cmode, int errLevel = 1) ;
    /** Returns true if context file for given step and version is available */
    bool testContextFile (int stepNumber, int stepVersion);
    /**
    Creates new DataReader for given domain.
    Returns nonzero on success.
    @param domainNum domain number

```

```

@param domainSerNum domain seerial number
@param cmode determines the i/o mode of context file
*/
DataReader*      GiveDomainDataReader (int domainNum, int domainSerNum,
                                      ContextFileMode cmode) ;

/**
Updates components mapped to numerical method if necessary during solution process.
Some numerical methods may require updating
mapped components during solution process (e.g., updating of tanget stiffness
when using updated Newton-Raphson method).
@param tStep time when component is updated.
@param cmpn Numerical component to update.
*/
virtual void      updateComponent (TimeStep* tStep, NumericalCmpn cmpn);

/**
Initializes solution of new time step. Default implementation
resets all internal history variables (in integration points of elements)
to previously reached equilibrium values.
Can be used for time step restart.
*/
virtual void      initStepIncrements();

/** Forces equation renumbering on given domain. All equation numbers
in all dofManagers are invalidated, and new equation numbers are
generated starting from domainNeqs entry corresponding to given
domain. It will update numberOfEquations variable accordingly.
Should be used at startup to force equation numbering and therefore
sets numberOfEquations. Must be used if model supports changes of
static system to assign new valid equation numbers to dofManagers.
*/
virtual int      forceEquationNumbering (int i);

/** Forces equation renumbering on all domains associated to engng
model. All equation numbers in all domains for all dofManagers are
invalidated, and new equation numbers are generated starting from 1
on each domain. It will update numberOfEquations variable
accordingly. Should be used at startup to force equation numbering
and therefore sets numberOfEquations. Must be used if model supports
changes of static system to assign new valid equation numbers to
dofManagers. */
virtual int      forceEquationNumbering ();

/** Indicates if Engngmodel requires Dofs dictionaries to be updated.
If EngngModel does not support changes of static system, the dof
frowards the requests for its unknowns to EngngModel, where unknowns
are naturally kept. This is posible, because dof equation number is
same during whole solution. But when changes of static system are
allowed, several problem arise. For example by solving simple
incremental static with allowed static changes, the incremetal
displacement vector of structure can not be added to total
displacement vector of structure, because equation numbers may have
changed, and one can not simply add these vector to obtain new total
displacement vector, because uncompatible displacement will be added.
To solve this problem, uknown dictionary at dof level has been

```

```

assumed. Dof then keeps its unknowns in its own private dictionary.
After computing increment of solution, engngModel updates for each
dof its unknowns in its dictionary (using updateUnknownsDictionary
function). For aforementioned example engngModel updates incremental
values but also total value by asking dof for previous total value
(dof will use its dictionary, does not ask back EngngModel) adds
corresponding increment and updates total value in dictionary.
*/
virtual int      requiresUnknownsDictionaryUpdate () {return 0;}

/** Returns true if equation renumbering is required for given
solution step. This may of course change the number of equation and
in general there is no guarantee that for a certain dof the same
equation will be assigned. So the use of DOF unknowns dictionaries
is generally recommended. */
virtual bool requiresEquationRenumbering(TimeStep*) {return false;}
//virtual int      supportsBoundaryConditionChange () {return 0;}

/**
Updates necessary values in Dofs unknown dictionaries.
@see EngngModel::requiresUnknownsDictionaryUpdate
@see Dof::updateUnknownsDictionary
*/
virtual void      updateDofUnknownsDictionary (DofManager*, TimeStep*) {}

/** This method is responsible for computing unique dictionary id
(ie hash value) from given equationId, valueModeType and
timestep. This function is used by particular dofs to access unknown
identified by given params from its dictionary using computed index.
Usually the hash algorithm should produce index that depend on
timestep relatively to actual one to avoid storage of complete
history.
*/
virtual int giveUnknownDictHashIndx (EquationID type, ValueModeType mode,
TimeStep* stepN) {return 0;}

// we don't directly call element ->GiveCharacteristicMatrix() function, because some
// engngm classes may require special modification of base types supported on
// element class level

/** Returns characteristic matrix of element. The
Element::GiveCharacteristicMatrix function should not be called
directly, because EngngModel may require some special modification of
characteristic matrices supported on element level. But default
implementation does the direct call to element level.

@param answer characteristic matrix
@param num element number
@param type type of CharMatrix requested
@param tStep time step when response is computed
@param domain source domain
*/
virtual void giveElementCharacteristicMatrix (FloatMatrix& answer, int num,
CharType type, TimeStep* tStep, Domain* domain)

```

```

    { domain->giveElement(num)->giveCharacteristicMatrix (answer, type, tStep);}

/** Returns characteristic vector of element. The
Element::GiveCharacteristicVector function should not be called
directly, because EngngModel may require some special modification of
characteristic vectors supported on element level. But default
implementation does the direct call to element level.

@param answer characteristic vector
@param num element number
@param type type of vector requested
@param tStep time step when response is computed
@param domain source domain
*/
virtual void giveElementCharacteristicVector (FloatArray& answer, int num,
        CharType type, ValueModeType mode, TimeStep* tStep, Domain* domain)
{ domain->giveElement(num)->giveCharacteristicVector (answer, type, mode, tStep);}

protected:
/**
    Assembles characteristic matrix of required type into given sparse matrix.
    @param answer assembled matrix
    @param tStep time step, when answer is assembled.
    @param ut determines type of equation and corresponding element code numbers
    @param type characteristic components of type type are requested from elements and assembled.
    @param domain source domain
*/
virtual void          assemble (SparseMtrx *answer, TimeStep* tStep, EquationID ut,
        CharType type, Domain* domain) ;

/**
    Assembles characteristic matrix of required type into given sparse matrix.
    @param answer assembled matrix
    @param tStep time step, when answer is assembled.
    @param r_id determines type of equation and corresponding element code numbers for matrix rows
    @param c_id determines type of equation and corresponding element code numbers for matrix columns
    @param type characteristic components of type type are requested from elements and assembled.
    @param domain source domain
*/
virtual void assemble (SparseMtrx *answer, TimeStep* tStep, EquationID r_id,
        EquationID c_id, CharType type, Domain* domain) ;

/**
    Assembles characteristic vector of required type into given vector.
    @param answer assembled vector
    @param tStep time step, when answer is assembled.
    @param type characteristic components of type type are requested
    from dofManagers/elements and assembled.
*/
//virtual void          assemble (FloatArray&, TimeStep*, CharType type,
        Domain* domain) ;

/**
    Assembles characteristic vector of required type from dofManagers into given vector.
    @param answer assembled vector
    @param tStep time step, when answer is assembled.
    @param type characteristic components of type type are requested

```



```

    from dofManagers and assembled using code numbers.
*/
virtual void assembleVectorFromDofManagers (FloatArray&, TimeStep*, EquationID ut,
                                           CharType type, ValueModeType mode,
                                           Domain* domain) ;

/** Assembles prescribed characteristic vector of required type
    from dofManagers into given vector.

    @param answer assembled vector
    @param tStep time step, when answer is assembled.
    @param type characterisctic components of type type are requested
    from dofManagers and assembled using prescribed eqn numbers.
*/
void assemblePrescribedVectorFromDofManagers (FloatArray&, TimeStep*, EquationID,
                                              CharType type, ValueModeType mode,
                                              Domain* domain) ;

/**
    Assembles characteristic vector of required type from elements into given vector.
    @param answer assembled vector
    @param tStep time step, when answer is assembled.
    @param type characterisctic components of type type are requested
    from elements and assembled using using code numbers.
*/
void assembleVectorFromElements (FloatArray&, TimeStep*, EquationID,
                                CharType type, ValueModeType mode,
                                Domain* domain) ;

/**
    Assembles prescribed characteristic vector of required type from
    elements into given vector.
    @param answer assembled vector
    @param tStep time step, when answer is assembled.
    @param type characterisctic components of type type are requested
    from elements and assembled using prescribed eqn numbers.
*/
void assemblePrescribedVectorFromElements (FloatArray&, TimeStep*, EquationID,
                                           CharType type, ValueModeType mode, Domain* domain) ;

public:

    // consistency check

    /** Allows programmer to test some receiver's internal data, before
        computation begins.
        @return nonzero if receiver check is o.k. */
    virtual int checkConsistency () {return 1;} // returns nonzero if o.k.
    /** Allows programmer to test problem its internal data, before computation begins.
        @return nonzero if receiver check is o.k. */
    int checkProblemConsistency (); // returns nonzero if o.k.

/**
    Prints output of receiver to ouput domain stream, for given time step.
    Corresponding function for element gauss points is invoked
    (gaussPoint::printOutputAt).
```

```

*/
virtual void                printOutputAt (FILE *, TimeStep*) ;

// input / output
/// Prints state of receiver. Useful for debugging.
void printYourself () ;

/** DOF printing routine. Called by DofManagers to print Dof specific part.
Dof class provides component printing routines, but emodel is responsible
for what will be printed at DOF level.
@param stream output stream
@param iDof dof to be processed
@param atTime solution step
*/
virtual void printDofOutputAt (FILE* stream, Dof* iDof, TimeStep* atTime) = 0;

// identification
/// Returns class name of the receiver.
virtual const char* giveClassName () const { return "EngngModel" ;}
/// Returns classType id of receiver.
virtual classType giveClassID () const { return EngngModelClass ;}
/// Returns nonzero if receiver does incremental analysis.
virtual int isIncremental () {return 0;}
/// Returns nonzero if nonlocal stiffness option activated.
virtual int useNonlocalStiffnessOption () {return 0;}
/// return true if receiver in parallel mode
bool isParallel () {return (parallelFlag != 0);}

/**
Indicates type of non linear computation (total or updated formulation).
This is used for example on Nodal level to update coordinates
if updated formulation
is done, or on element level, when non linear contributions are computed.
*/
virtual fMode giveFormulation () {return UNKNOWN;} // for non-linear computation
/*
Returns Load Response Mode of receiver.
This value indicates, whether nodes and elements should assemble
total or incremental load vectors.

virtual LoadResponseMode giveLoadResponseMode () {return TotalLoad;}
*/
/// Context requesting service
EngngModelContext* giveContext () {return this->context;}
/**
Returns number of slave problems */
virtual int giveNumberOfSlaveProblems() {return 0;}
/**Returns i-th slave problem */
virtual EngngModel* giveSlaveProblem (int i) {return NULL;}

/** Returns the Equation scaling flag, which is used to indicate
that governing equation(s) are scaled, or non-dimensionalized */

```

```

virtual bool giveEquationScalingFlag () {return false;}
/// Returns the scale factor for given variable type
virtual double giveVariableScale (VarScaleType varId) {return 1.0;}

/**@name error and warning reporting methods These methods will
print error (or warning) message using oofem default loggers. Do
not use these methods directly, to avoid specify file and line
parameters. More preferably, use these methods via corresponding
OOFEM_CLASS_ERROR and OOFEM_CLASS_WARNING macros, that will include
file and line parameters automatically. Uses variable number of
arguments, so a format string followed by optional arguments is
expected (according to printf conventions).
@param file source file name, where error encountered (where error* function called)
@param line source file line number, where error encountered
*/
//@{
/// prints error message and exits
void error (char* file, int line, char *format, ...) const ;
/// prints warning message
void warning (char* file, int line, char *format, ...) const ;
//@}

};

typedef EngngModel Problem;

```

The key method declared by **EngngModel** is *solveYourself*, which starts the solution. It loops over all metasteps. For each metastep, the loop over corresponding solution steps is performed. For each solution step, the value of *currentStep* attribute is updated by invoking *giveNextStep()* service, and the *solveYourselfAt* is called, performing the solution for given step. The *currentStep* is an attribute of *EngngModel* class. At the very beginning, it is set to NULL or is initialized to the step from which analysis is restarted.

```

void
EngngModel :: solveYourself ()
{
    int imstep, jstep;
    int smstep=1, sjstep=1;
    MetaStep* activeMStep;
#ifdef TIME_REPORT
    oofem_timeval tstart;
#endif

    // restart support - if currentStep is set already, start from the
    // next one
    if (this->currentStep) {
        smstep = this->currentStep->giveMetaStepNumber();
        sjstep = this->giveMetaStep(smstep)->
            giveStepRelativeNumber(this->currentStep->giveNumber()) + 1;
    }
}

```

```

    for (imstep = smstep; imstep<= nMetaSteps; imstep++) {
        activeMStep = this->giveMetaStep(imstep);
        for (jstep = sjstep; jstep <= activeMStep->giveNumberOfSteps(); jstep++)
        {
#ifdef TIME_REPORT
            ::getUtime(tstart);
#endif
            this->giveNextStep();
            // update attributes according to new meta step attributes
            if (jstep == sjstep) this->updateAttributes (this->giveCurrentStep());

            this->solveYourselfAt(this->giveCurrentStep());

#ifdef TIME_REPORT
            oofem_timeval ut;
            ::getRelativeUtime (ut, tstart);
            printf ("\nEngngModel info: user time consumed by solution step %d: %.2fs\n",
                    jstep, (double)(ut.tv_sec+ut.tv_usec/(double)OOFEM_USEC_LIM));
#endif
        }
    }
}

```

The *solveYourselfAt* typically assembles characteristic matrices and vectors and solve the problem using the suitable numerical method. After finishing the solution, the *updateYourself* service for updating solution state and then *terminate* method (for updating nodal and element values) should be called to consistently update the state of all components. The implementation should be provided by derived classes (see section 3.4 for an example).

The implementation of *updateYourself* service loops over all problem domains and calls corresponding update service for all DOF managers and elements. The *terminate* service prints the required outputs and optionally saves the context file (if required), so the solution can be restarted from this saved state later. Both services are virtual, so they can be easily tailored to specific needs.

```

void
EngngModel :: updateYourself (TimeStep* stepN)
{
    int idomain, ndomains = this->giveNumberOfDomains();
    int j, nnodes;
    Domain* domain;

    for (idomain = 1; idomain <= ndomains; idomain++) {

        domain= this->giveDomain(idomain);

#ifdef VERBOSE
        VERBOSE_PRINT0("Updating domain ",domain->giveNumber())
#endif

        nnodes = domain->giveNumberOfDofManagers ();
        for( j=1;j<=nnodes;j++) {
            domain->giveDofManager(j)->updateYourself(stepN) ;
        }
    }
}

```

```

# ifdef VERBOSE
  VERBOSE_PRINT0("Updated nodes & sides ",nnodes)
# endif

  Element* elem;

  int nelem = domain->giveNumberOfElements ();
  for (j=1 ; j<=nelem ; j++) {
    elem = domain -> giveElement(j) ;
#ifdef __PARALLEL_MODE
    // skip remote elements (these are used as mirrors
    // of remote elements on other domains, when nonlocal
    // constitutive models are used.
    // Their introduction is necessary to allow local
    // averaging on domains without fine grain
    // communication between domains)

    if (elem->giveParallelMode () == Element_remote) continue;
#endif
    elem -> updateYourself(stepN);
  }

# ifdef VERBOSE
  VERBOSE_PRINT0("Updated Elements ",nelem)
# endif

}
}

void
EngngModel :: terminate (TimeStep* stepN)
{
  FILE* File = this->giveOutputStream();
  // print output
  this->printOutputAt (File, stepN);

  // save context if required
  // default - save only if ALWAYS is set ( see cltypes.h )

  if ((this->giveContextOutputMode() == ALWAYS) ||
      (this->giveContextOutputMode() == REQUIRED)) {
    this->saveContext(NULL);
  } else if (this->giveContextOutputMode() == USERDEFINED) {
    if (stepN->giveNumber()%this->giveContextOutputStep() == 0)
      this->saveContext(NULL);
  }
}
}

```

The implementations of services for characteristic components assembly are provided. They simply loop over nodes or elements (depending on the character of the requested component) of the given domain, requesting the corresponding component contributions and corresponding code numbers. The component contributions are assembled (using code numbers) into a target array or matrix. The implementation of *assemble* for characteristic vectors has to determine whether the contribution comes from node or element. The implementation presented here uses the hard-wired decision

rules (adapted for structural analysis), which in other cases leads the overloading of basic implementation. The better solution will be probably to call some virtual service, which returns the source of contribution (nodal or element contribution) and then to perform corresponding loop.

```
void EngngModel :: assemble (SparseMtrx* answer, TimeStep* tStep, EquationID ut,
                             CharType type, Domain* domain)
//
// assembles matrix answer by calling
// element(i) -> giveCharacteristicMatrix ( type, tStep );
// for each element in domain
// and assembling every contribution to answer
//
//
{
    int ielem;
    IntArray loc ;
    FloatMatrix mat;
    Element *element;

    if (answer == NULL) _error("assemble: NULL pointer encountered.");

    int nelem = domain -> giveNumberOfElements ();
    for ( ielem = 1; ielem <= nelem ; ielem++ ) {
        element = domain -> giveElement(ielem);
#ifdef __PARALLEL_MODE
        // skip remote elements (these are used as mirrors of remote elements on other domains
        // when nonlocal constitutive models are used. Their introduction is necessary to
        // allow local averaging on domains without fine grain communication between domains).
        if (element->giveParallelMode () == Element_remote) continue;
#endif
        element -> giveLocationArray (loc, ut);
        this->giveElementCharacteristicMatrix(mat, ielem, type, tStep, domain );

        if (mat.isEmpty()) {
            if (answer -> assemble (loc, mat) == 0)
                _error("assemble: sparse matrix assemble error");
        }
    }
    answer->assembleBegin();
    answer->assembleEnd();
}

void EngngModel :: assembleVectorFromDofManagers
(FloatArray& answer, TimeStep* tStep, EquationID ut,
 CharType type, ValueModeType mode, Domain* domain)
//
// assembles matrix answer by calling
// node(i) -> computeLoadVectorAt (tStep);
// for each element in domain
// and assembling every contribution to answer
//
//
{
    int i ;
```

```

IntArray loc ;
FloatArray charVec ;
DofManager *node ;

int nnode = domain -> giveNumberOfDofManagers();

for (i = 1; i <= nnode ; i++ ) {
    node = domain -> giveDofManager(i);
    node -> computeLoadVectorAt (charVec, tStep, mode);
    if(charVec.giveSize()) {
        node -> giveCompleteLocationArray (loc);
        answer.assemble (charVec, loc) ;
    }
}

void EngngModel :: assembleVectorFromElements
    (FloatArray& answer, TimeStep* tStep, EquationID ut,
     CharType type, ValueModeType mode, Domain* domain)
//
// assembles matrix answer by calling
// element(i) -> giveCharacteristicMatrix ( type, tStep );
// for each element in domain
// and assembling every contribution to answer
//
//
{
    int i ;
    IntArray loc ;
    FloatArray charVec ;
    Element *element ;

    int nelem = domain -> giveNumberOfElements ();

    for (i = 1; i <= nelem ; i++ ) {
        element = domain -> giveElement(i);
#ifdef __PARALLEL_MODE
        // skip remote elements (these are used as mirrors of remote elements on other domains
        // when nonlocal constitutive models are used. Their introduction is necessary to
        // allow local averaging on domains without fine grain communication between domains).
        if (element->giveParallelMode () == Element_remote) continue;
#endif
        element -> giveLocationArray (loc, ut);
        this -> giveElementCharacteristicVector (charVec, i, type, mode, tStep, domain);
        if(charVec.giveSize()) answer.assemble (charVec, loc) ;
    }
}

void EngngModel :: assemblePrescribedVectorFromDofManagers
    (FloatArray& answer, TimeStep* tStep, EquationID ut,
     CharType type, ValueModeType mode, Domain* domain)
//
// assembles matrix answer by calling
// node(i) -> computeLoadVectorAt (tStep);

```

```

// for each element in domain
// and assembling every contribution to answer
//
//
{
    int i ;
    IntArray loc ;
    FloatArray charVec ;
    DofManager *node ;

    int nnode = domain -> giveNumberOfDofManagers();

    for (i = 1; i <= nnode ; i++ ) {
        node = domain -> giveDofManager(i);
        node -> computeLoadVectorAt (charVec, tStep, mode);
        if(charVec.giveSize()) {
            node -> giveCompletePrescribedLocationArray (loc);
            answer.assemble (charVec, loc) ;
        }
    }
}

void EngngModel :: assemblePrescribedVectorFromElements
    (FloatArray& answer, TimeStep* tStep, EquationID ut,
     CharType type, ValueModeType mode, Domain* domain)
//
// assembles matrix answer by calling
// element(i) -> giveCharacteristicMatrix ( type, tStep );
// for each element in domain
// and assembling every contribution to answer
//
//
{
    int i ;
    IntArray loc ;
    FloatArray charVec ;
    Element *element ;

    int nelem = domain -> giveNumberOfElements ();

    for (i = 1; i <= nelem ; i++ ) {
        element = domain -> giveElement(i);
#ifdef __PARALLEL_MODE
        // skip remote elements (these are used as mirrors of remote eleemnts on other domains
        // when nonlocal constitutive models are used. They introduction is necessary to
        // allow local averaging on domains without fine grain communication between domains).
        if (element->giveParallelMode () == Element_remote) continue;
#endif
        element -> givePrescribedLocationArray (loc, ut);
        if (loc.containsOnlyZeroes()) continue;
        this -> giveElementCharacteristicVector (charVec, i, type, mode, tStep, domain);
        if(charVec.giveSize()) answer.assemble (charVec, loc) ;
    }
}

```

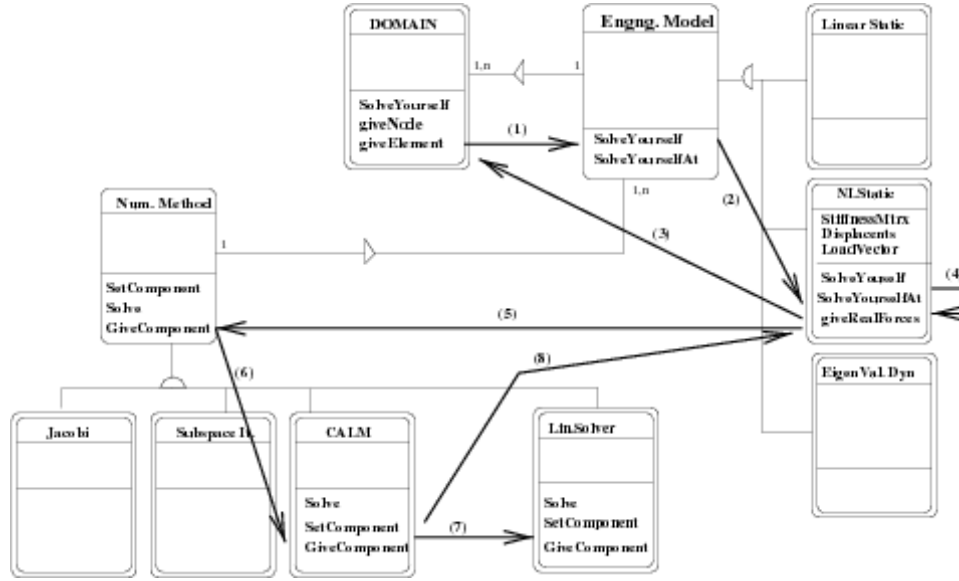



Figure 2: Engng model - Numerical method Interface.

3.2 Numerical Method Interface

In order to solve the governing equation, the instance of a suitable numerical method is created. Engng model may use different numerical methods, depending, for example, on problem size or previous convergence.

Typically, each particular Engng model instance is responsible for mapping of its governing equation components to corresponding numerical components. Such mapping allows the numerical method implementation to be independent of a particular physical problem by strictly dealing with numerical components, which are mapped to corresponding physical components of governing equation, that are hidden to numerical method.

Engng model must also provide services for updating its components, if this is necessary. These are used, when the **Numerical Method** instance needs to update some components during solution (for example in the Newton Raphson algorithm for the solution of non-linear equations, stiffness has to be updated after each iteration). Similarly, a high-level numerical method may use the services of another low-level numerical method (solver for non-linear system of equations uses linear solver for linearized problem). **Numerical method** instance may also represent an interface to a procedure in C or Fortran (see Fig. ??).

The derived classes from **Numerical method** are supposed to declare the interface for specific problem type (like solution of linear system). The interface usually consist in declaring virtual abstract function solve, with parameters corresponding to problem under consideration. The solve method should return value of NM.Status type. The data are specified using parameters passed to solve method (so called mapping). (Other optional parameters can be provided via instantiateYourself service, which receives the init record of corresponding metastep).

It should be pointed out, that all numerical methods solving the same numerical problem use the same general interface (same mapping) - this is enforced by using the same base problem-specific class. It is therefore possible to use any suitable instance of the Numerical method class to solve the problem, and leave the whole engineering model code, including mapping, unchanged, because all instances of the Numerical method class provide the common interface.

This concept is further enhanced by the introduction of a base abstract class for all sparse matrices. This class only declares the basic required services provided by all sparse matrices (like multiplication by a vector, possible factorization, etc). The implementation is left on derived classes. Numerical methods are then implemented only

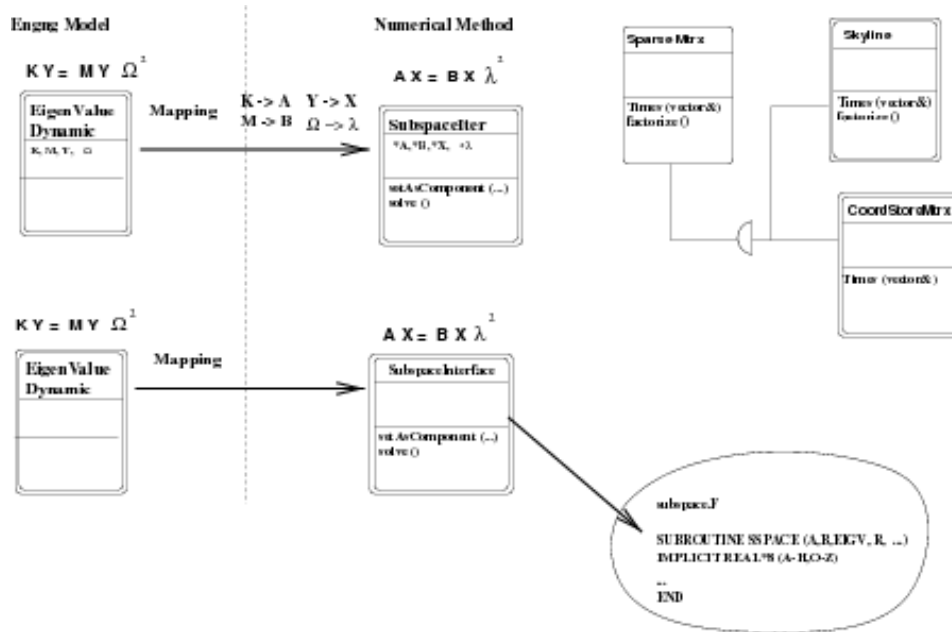


Figure 3: Engng model - Numerical method Interface.

using basic services declared by the **Sparse Matrix** class. Thus, numerical method class instances will work with any sparse matrix class, even those added in the future, without changing any code, because all derived classes of the **Sparse Matrix** class implement the same interface.

The declaration of **SparseLinearSystem** class (typical problem-specific base class defining the interface for solving sparse system of equations) is listed to illustrate the basic concepts.

```
// *****
// *** CLASS SparseLinearSystemNM ***
// *****

#ifdef sparselinsystemnm_h

#include "nummet.h"
#include "cltypes.h"
#include "typeInfo.h"
#include <stdio.h>
class EngngModel ; class SparseMtrx; class FloatArray;

/**
This base class is an abstraction for all numerical methods
solving sparse linear system of equations.
The purpose of this class is to declare the general interface
to all numerical methods solving this kind of problem.
This interface allows to use any suitable instance of the
Numerical method class to the solve problem,
and leave the whole engineering model code,
```

```

including mapping, unchanged, because all instances of this class
provide the common interface.
*/
class SparseLinearSystemNM : public NumericalMethod
{
protected:
public:
    /// Constructor
    SparseLinearSystemNM (int i, Domain* d, EngngModel* m);
    /// Destructor
    ~SparseLinearSystemNM ();

    // identification
    /// Returns class name of the receiver.
    char* giveClassName (char* s) const
    { return strcpy(s, "SparseLinearSystemNM") ;}
    /** Returns classType id of receiver.
    @see FEMComponent::giveClassID
    */
    classType giveClassID () const { return SparseLinearSystemNMClass ;}

    /**
    Solves the given sparse linear system of equations Ax=b.
    @param A coefficient matrix
    @param b right hand side
    @param x solution array
    @return NM_Status value
    */
    virtual NM_Status solve (SparseMtrx* A, FloatArray* b, FloatArray* x) = 0;

public:
};

#define sparselinsystemnm_h
#endif

```

To summarize, the natural independence of the problem formulation, numerical solution of the problem, and data storage format have been obtained, which leads to a modular and extensible structure of the engineering model - numerical method frame.

3.3 Grid Representation - Domain

The computational grid is represented by **Domain** class. It stores all the components of the FEM model. These are dof managers (nodes, element sides possessing DOFs), elements, material and cross section models, boundary conditions (primary boundary conditions as well as applied loading), and initial conditions. For every component type the Domain maintains the component list and provides the corresponding access services. Domain keeps its number and link to associated problem (EngngModel). It also keeps its domain type, which determines the default number of Dofs per node (or side) and their physical meanings. Note, that this default setting can be redefined by particular nodes or sides. The basic services provided by Domain are the following

- Reading its description from input and creating corresponding objects. This task includes the reading and parsing the particular mesh input records, creating the corresponding components representations (objects) of

appropriate type, initializing these components using their *instanciteFromString* methods and storing them into corresponding list.

- Provides services for accessing its particular components. The services returning the total number of particular domain components and particular component access methods based on component number are provided.

Domain also contains instance of **OutputManager** manager class, which implements the output filtering capabilities. It allows to filter output for specific solution steps, elements and dof managers. The domain also can create (if needed) instances of **SpatialLocalizer** class and **connectivityTable** class to serve the connectivity and spatial localization related services (finding elements shared by the node, finding the closest node search, finding the element containing given point, etc.).

3.4 Example - Linear Static Analysis

In this section, the example of the implementation of linear static analysis will be shown. The meta steps are not used, so the default one is created. The class definition includes the declaration of characteristic components of the problem - the stiffness matrix and load and displacement vectors. Two additional variables are used to store the solver type and the sparse matrix type, which can be selected by the user. The services for solving the solution step (*solveYourselfAt*), requesting unknowns (*giveUnknownComponent*), context i/o (*saveContext* and *restoreContext*), problem initialization (*initializeFrom*), and consistency checking (*checkConsistency*) are overloaded.

```
class LinearStatic : public StructuralEngngModel
{
/*
    This class implements LinearStatic Engineering problem.
    Multiple loading works only if linear elastic material (such as isoLE) is used.
    (Other non-linear materials kepp load history, so such multiple loading
    will cause that next step will be assumed as new load increment,
    not the total new load). Because they always copute real stresses according
    to reached strain state, they are not able to respond to linear analysis.
```

DESCRIPTION:

Solution of this problem is series of loading cases, maintained as sequence of time-steps. This solution is in form of linear equation system $Ax=b$

TASK:

Creating Numerical method for solving $Ax=b$
 Interfacing Numerical method to Elements
 Managing time steps

*/

```
protected:
    SparseMtrx* stiffnessMatrix;
    FloatArray loadVector;
    FloatArray displacementVector;

    LinSystSolverType solverType;
    SparseMtrxType sparseMtrxType;
    /// Numerical method used to solve the problem
    SparseLinearSystemNM *nMethod;

    int initFlag;

#ifdef __PETSC_MODULE
    Vec _loadVec, _dispVec;
```

```

#endif

public:
    LinearStatic (int i, EngngModel* _master = NULL) ;
    ~LinearStatic ()
    {delete stiffnessMatrix; if (nMethod) delete nMethod;}
// solving
    void solveYourself ();
    void solveYourselfAt (TimeStep *);
    //int requiresNewLhs () {return 0;}
/**
    Updates nodal values
    (calls also this->updateDofUnknownsDictionary for updating dofs unknowns dictionaries
    if model supports changes of static system). The element internal state update is also forced using
    updateInternalState service.
    */
    virtual void          updateYourself (TimeStep*) ;
    double   giveUnknownComponent ( EquationID, ValueModeType, TimeStep*, Domain*, Dof*);
    contextIOResultType saveContext (FILE* stream, void *obj = NULL) ;
    contextIOResultType restoreContext (FILE* stream, void *obj = NULL);

void   updateDomainLinks();

    TimeStep* giveNextStep ();
    NumericalMethod* giveNumericalMethod (TimeStep*);
    //void printReactionForces (TimeStep *);
    void          terminate (TimeStep*);

IRResultType initializeFrom (InputRecord* ir);

// consistency check
virtual int checkConsistency (); // returns nonzero if o.k.

/** DOF printing routine. Called by DofManagers to print Dof specific part.
Dof class provides component printing routines, but emodel is responsible
for what will be printed at DOF level.
@param stream output stream
@param iDof dof to be processed
@param atTime solution step
    */
virtual void printDofOutputAt (FILE* stream, Dof* iDof, TimeStep* atTime);

// identification
    const char* giveClassName () const { return "LinearStatic";}
    classType giveClassID ()      const { return LinearStaticClass;}
    fMode giveFormulation () { return TL; }
#ifdef __PARALLEL_MODE
/**
    Determines the space necessary for send/receive buffer.
    It uses related communication map pattern to determine the maximum size needed.
    @param commMap communication map used to send/receive messages
    @param buff communication buffer
    @return upper bound of space needed
    */

```

```

    int estimateMaxPackSize (IntArray& commMap, CommunicationBuffer& buff, int packUnpackType) ;
#endif

} ;

```

The listing of the **LinearStatic** class implementation follows. First, the *giveNumericalMethod* returns the numerical method, which will be used. The new instance is created if not defined according to *nMethod* attribute, which is initialized from input (see *instanciateYourself*). Note, that thanks to the general interface, defined by the **NumericalMethod** class and the component mapping approach, the further communication with the numerical method uses only the interface declared by the base **NumericalMethod** class and therefore there is no need to worry about particular numerical method details.

```

NumericalMethod* LinearStatic :: giveNumericalMethod (TimeStep* atTime)
// only one has reason for LinearStatic
//      - SolutionOfLinearEquations

{
    if (nMethod) return nMethod ;

    SparseLinearSystemNM* nm;
    if (solverType == ST_Direct) {
        nm = (SparseLinearSystemNM*) new LDLTFactorization (1,this->giveDomain(1),this);
        nMethod = nm;
        return nm;
    } else {
        nm = (SparseLinearSystemNM*) new IMLSolver (1,this->giveDomain(1),this);
        nMethod = nm;
        return nm;
    }
}

IRResultType
LinearStatic :: initializeFrom (InputRecord* ir)
{
    // Required by IR_GIVE_FIELD macro
    const char *__keyword, *__proc = "initializeFrom";
    IRResultType result;

    StructuralEngngModel::initializeFrom (ir);
    int val = 0;
    IR_GIVE_OPTIONAL_FIELD (ir, val, "lstype"); // Macro
    solverType = (LinSystSolverType) val;

    val = 0;
    IR_GIVE_OPTIONAL_FIELD (ir, val, "smttype"); // Macro
    sparseMtrxType = (SparseMtrxType) val;

    return IRRT_OK;
}

```

```

double LinearStatic :: giveUnknownComponent (EquationID chc, ValueModeType mode,
      TimeStep* tStep, Domain* d, Dof* dof)

```

```

// returns unknown quantity like displacement, velocity of equation eq
// This function translates this request to numerical method language
{

    int eq = dof->giveEquationNumber();
    if (eq == 0) _error ("giveUnknownComponent: invalid equation number");

    if (tStep != this->giveCurrentStep ()) {
        _error ("giveUnknownComponent: unknown time step encountered");
        return 0.;
    }

    if (chc != EID_MomentumBalance) {
        _error ("giveUnknownComponent: Unknown is of undefined CharType for this problem");
        return 0.;
    }

    switch (mode)
    {
        case VM_Total:
        case VM_Incremental:
            if (displacementVector.isEmpty()) return displacementVector.at(eq);
            else return 0.;
            // return nMethod-> giveUnknownComponent (LinearEquationSolution, eq);

        default:
            _error ("giveUnknownComponent: Unknown is of undefined type for this problem");
    }
    return 0.;
}

TimeStep* LinearStatic :: giveNextStep ()
{
    int istep = this->giveNumberOfFirstStep();
    //int mstep = 1;
    StateCounterType counter = 1;
    delete previousStep;

    if (currentStep != NULL) {
        istep = currentStep->giveNumber() + 1 ;
        counter = currentStep->giveSolutionStateCounter() + 1;
    }
    previousStep = currentStep;
    currentStep = new TimeStep (istep,this, 1, (double) istep, 0., counter);
    // time and dt variables are set eq to 0 for staics - has no meaning
    return currentStep;
}

void LinearStatic :: solveYourselfAt (TimeStep* tStep) {
    //
    // creates system of governing eq's and solves them at given time step
    //
    // first assemble problem at current time step

```

```

    if (initFlag) {
#ifdef VERBOSE
        OOFEM_LOG_INFO("Assembling stiffness matrix\n");
#endif

        //
        // first step assemble stiffness Matrix
        //
        /*
            IntArray* mht = this -> GiveBanWidthVector ();
            stiffnessMatrix = new Skyline ();
            stiffnessMatrix -> checkSizeTowardsBanWidth (mht) ;
            delete mht;
        */
        stiffnessMatrix = ::CreateUsrDefSparseMtrx(sparseMtrxType); // new Skyline ();
        if (stiffnessMatrix==NULL) _error ("solveYourselfAt: sparse matrix creation failed");
        //stiffnessMatrix = new DynCompCol ();
        //stiffnessMatrix = new CompCol ();

        stiffnessMatrix->buildInternalStructure (this, 1, EID_MomentumBalance);

        this -> assemble (stiffnessMatrix, tStep, EID_MomentumBalance, StiffnessMatrix, this->giveDomain(1));

        //
        // allocate space for displacementVector
        //
        //displacementVector = new FloatArray (this->giveNumberOfEquations());
        displacementVector.resize (this->giveNumberOfEquations(EID_MomentumBalance));
        displacementVector.zero();

#ifdef __PETSC_MODULE
        if (solverType == ST_Petsc) {
            this->givePetscContext(1, EID_MomentumBalance)->createVecGlobal (&_loadVec);
            this->givePetscContext(1, EID_MomentumBalance)->createVecGlobal (&_dispVec);
        }
#endif
        initFlag = 0;
    }
#ifdef VERBOSE
        OOFEM_LOG_INFO("Assembling load\n");
#endif

#ifdef __PETSC_MODULE
        // direct interface to PETSC
        if (solverType == ST_Petsc) {
            this->petsc_assembleVectorFromElements(_loadVec, tStep, EID_MomentumBalance,
                                                ElementForceLoadVector, VM_Total,
                                                this->giveDomain(1));
            this->petsc_assembleVectorFromElements(_loadVec, tStep, EID_MomentumBalance,
                                                ElementNonForceLoadVector, VM_Total,
                                                this->giveDomain(1));
            this->petsc_assembleVectorFromDofManagers(_loadVec, tStep, EID_MomentumBalance,

```



```

                                NodalLoadVector, VM_Total,
                                this->giveDomain(1)) ;

    VecAssemblyBegin(_loadVec);
    VecAssemblyEnd (_loadVec);
    this->giveNumericalMethod(tStep);
#ifdef VERBOSE
    OOFEM_LOG_INFO("Solving ...\\n");
#endif

    //nMethod -> solveYourselfAt(tStep);
    PetscSolver *ps = dynamic_cast<PetscSolver*>(nMethod);
    PetscSparseMtrx* psm = dynamic_cast<PetscSparseMtrx*>(stiffnessMatrix);
    ps -> petSc_solve (psm, _loadVec, _dispVec);

    this->givePetscContext(1, EID_MomentumBalance)->scatterG2N (_dispVec, &displacementVector, INSERT_VALUES);
} else
#endif
{
    //
    // assembling the element part of load vector
    //
    //loadVector = new FloatArray (this->giveNumberOfEquations());
    loadVector.resize (this->giveNumberOfEquations(EID_MomentumBalance));
    loadVector.zero();

    this->assembleVectorFromElements(loadVector, tStep, EID_MomentumBalance,
                                    ElementForceLoadVector, VM_Total, this->giveDomain(1));
    this->assembleVectorFromElements(loadVector, tStep, EID_MomentumBalance,
                                    ElementNonForceLoadVector, VM_Total, this->giveDomain(1));

    //
    // assembling the nodal part of load vector
    //
    this->assembleVectorFromDofManagers(loadVector, tStep, EID_MomentumBalance,
                                        NodalLoadVector, VM_Total, this->giveDomain(1)) ;

    //
    // set-up numerical model
    //
    this->giveNumericalMethod(tStep);

    /*
    nMethod -> setSparseMtrxAsComponent ( LinearEquationLhs , stiffnessMatrix) ;
    nMethod -> setFloatArrayAsComponent ( LinearEquationRhs , &loadVector) ;
    nMethod -> setFloatArrayAsComponent ( LinearEquationSolution, &displacementVector) ;
    */
    //
    // call numerical model to solve arised problem
    //
#ifdef VERBOSE
    OOFEM_LOG_INFO("Solving ...\\n");
#endif

    //nMethod -> solveYourselfAt(tStep);

```

```

    nMethod -> solve (stiffnessMatrix, &loadVector, &displacementVector);
}
tStep->incrementStateCounter();           // update solution state counter
//
// update nodes, elements, etc.
this->updateYourself(this->giveCurrentStep());
}

```

```

void    LinearStatic :: updateYourself (TimeStep* stepN)
{
this->updateInternalState(stepN);
StructuralEngngModel::updateYourself(stepN);
}

```

```

contextIOResultType LinearStatic :: saveContext (FILE* stream, void *obj)
//
// saves state variable - displacement vector
//
{
contextIOResultType iores;
int closeFlag = 0;

if (stream==NULL) {
if (!this->giveContextFile(&stream,this->giveCurrentStep()->giveNumber(),
                        contextMode_write))
THROW_CIOERR(CIO_IOERR); // override
closeFlag = 1;
}

if ((iores = StructuralEngngModel :: saveContext (stream)) != CIO_OK)
    THROW_CIOERR(iores);
if ((iores = displacementVector.storeYourself(stream)) != CIO_OK)
    THROW_CIOERR(iores);

if (closeFlag) fclose (stream); // ensure consistent records
return CIO_OK;
}

```

```

contextIOResultType LinearStatic :: restoreContext (FILE* stream, void *obj)
//
// restore state variable - displacement vector
//
{
contextIOResultType iores;
int closeFlag = 0;
int istep = this->resolveCorrespondingStepNumber (obj);

if (stream == NULL) {
if (!this->giveContextFile(&stream, istep, contextMode_read))

```

```

    THROW_CIOERR(CIO_IOERR); // override
    closeFlag = 1;
}

if ((iores=StructuralEngngModel::restoreContext(stream,obj))!=CIO_OK)
    THROW_CIOERR(iores);
if ((iores=displacementVector.restoreYourself(stream))!=CIO_OK)
    THROW_CIOERR(iores);

    if (closeFlag) fclose (stream); // ensure consistent records
return CIO_OK;
}

void
LinearStatic:: terminate(TimeStep* tStep)
{
    StructuralEngngModel :: terminate (tStep);
    this->printReactionForces (tStep, 1);
}

int
LinearStatic::checkConsistency ()
{
    // check internal consistency
    // if success returns nonzero
    int i, nelem;
    Element* ePtr;
    StructuralElement* sePtr;
    Domain* domain = this->giveDomain(1);

    nelem = domain->giveNumberOfElements();
    // check for proper element type

    for (i=1; i<= nelem; i++) {
        ePtr = domain->giveElement(i);
        sePtr = DYNAMIC_CAST (StructuralElement, ePtr);
        if (sePtr == NULL) {
            printf ("Error: Element %d has no StructuralElement base\n",i);
            return 0;
        }
    }

    EngngModel :: checkConsistency ();

    return 1;
}

```

```

void
LinearStatic::updateDomainLinks ()
{
    EngngModel::updateDomainLinks();
    this->giveNumericalMethod(giveCurrentStep())->setDomain (this->giveDomain(1));
}

```

4 Material — Element Frame

As already mentioned, in Fig. 2 the material-element frame is schematically shown. In this frame, the following base classes & objects are introduced:

- Class **Element**, which is an abstraction of a finite element. It declares common general services, provided by all elements. Derived classes are the base classes for specific analysis types (structural analysis, thermal analysis). They declare and implement necessary services for specific analysis.
- **Integration point** class & object: It is an abstraction for the integration point of the finite element. It maintains its coordinates and integration weight. Any integration point can generally contain any number of other integration points - called slaves. The **Integration point** containing slaves is called master. Slaves are, for example, introduced by a layered cross section model, where they represent integration points for each layer, or can be introduced at material model level, where they may represent, for example, micro-planes. Slave integration points are hidden from elements. The **Integration point** also contains associated material status (the reasons for introducing this feature will be explained later).
- **Cross section** class is an abstraction for cross section. Its main role is to hide from an element all details concerning the cross section description and implementation. By cross section description is meant, for example, an integral cross section model, layered cross section model or fibered model. Elements do not communicate directly with material, instead they always use a **Cross Section** interface, which performs all necessary integration over its volume and invokes necessary material class services². The **Cross section** interface, defined in terms of general functions, allows the use of any cross section model, even that added in the future, without modification of any code, because all cross section models implement the same interface.
- **Material** class is shown here. It represents base class for all constitutive models. Derived classes should be the base analysis-specific classes, which declare required analysis specific services (for example structural material class declares services for the stiffness computation and services for the real stress evaluation). Again, the material analysis specific interface, defined in terms of general services, allows the use of any material model, even that added in the future, without modifying any code, because all material models implement the same interface.

One of the most important goals, which have been formulated, is extensibility. In the case of extension of the material library, the analyst is facing a key problem. Every material model must store its unique history parameters for every related integration point. The amount, type, and meaning of these history variables vary for each material model. Therefore, it is not possible to efficiently match all needs and to reflect them in the integration point data structure. The suggested remedy is the following:

The **Integration point** class is equipped with the possibility to have associated a **Material status** class. When a new material model is implemented, the analyst has also to declare and implement a related material status derived from the base **Material status** class to this material model. This status contains all necessary history variables and data access and modification services. The **Integration point** provides services for inserting and accessing its

²For some problems, the use of cross section is not necessary, and then the elements can communicate directly with material model. However, for some problems (for example structural analysis) the introduction of cross section is natural.

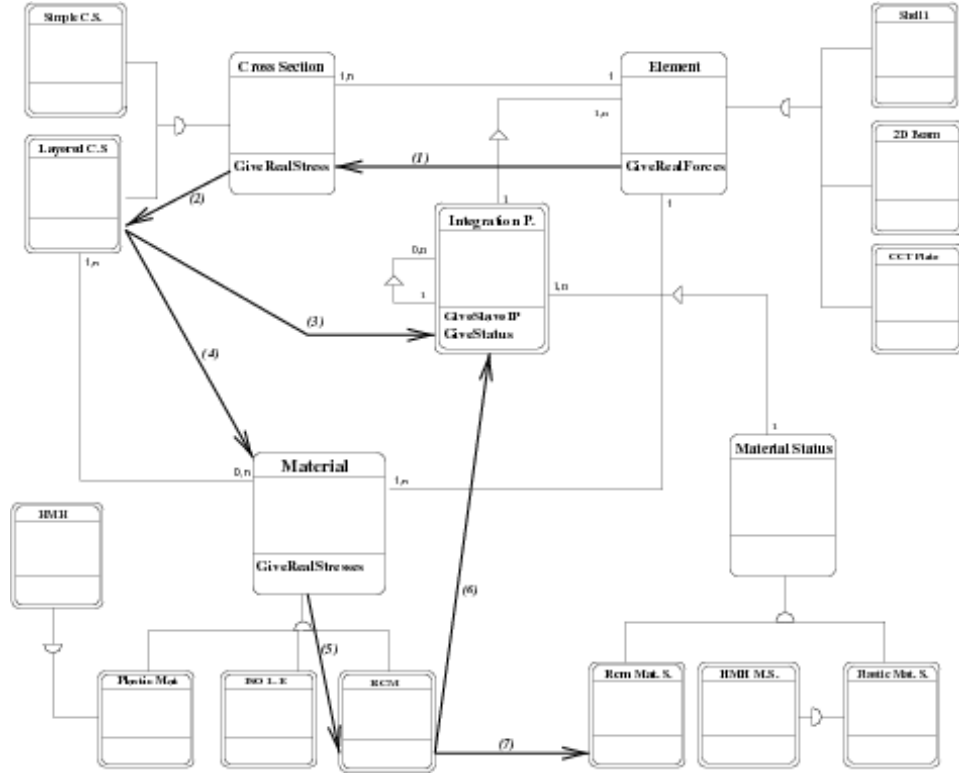


Figure 4: Element-material frame structure.

related status. For every **Integration point**, the corresponding material creates unique copy of its related material status and associates it with that integration point. Because the **Integration point** is a compulsory parameter of all messages sent to any material model, the particular material model can access its related material status from the given **Integration point**, and therefore can access its history variables.

In Fig. 4, the material - element frame is depicted in more detail, although it is still simplified. A simple cross section model class hierarchy is shown there. There are two derived classes from the parent **Cross Section** class: **Simple cross section** class representing an integral cross section model and **Layered cross section model** class, representing a layered cross section model implementation. At the bottom, the hierarchies of the material model and the associated material status representations are indicated. The program flow for an element, requesting the computation of its real nodal forces is also indicated in Fig. 4. This is generally done by integrating real stresses at its integration points. For each **Integration point** it asks the **Cross section** model to compute real stresses at the given integration point. In this example, the **Layered cross section** uses the master-slave integration point principle. Each element integration point, here called master, contains its slaves, each representing one layer. These slave integration points are introduced by the cross section model, and are hidden to the element. For a given master integration point, the cross section model performs integration over cross section volume using slaves. Therefore each slave integration point, which is requested by the master, uses material model class services to compute real stresses for each corresponding layer, passing the slave integration point as a parameter. Then for each slave, the material model asks the given integration point for its associated status. Having obtained reference to it, the material model can access all its history variables through status services, and computes results.

4.1 Base Element class

This class is the base class for all FE elements. It is also the abstract class, declaring some services, which have to be implemented by the derived classes. The main purpose of this class is to serve the basic common services, which are common to all finite elements (like storing the references to element's material model, nodes and applied loadings, assembling location array, storing from/to context file, etc.). The **Element** class does neither declare nor implement any method related to specific analysis. These services are to be declared and possibly implemented by derived classes.³ These derived classes (direct children of **Element** class) are therefore assumed to be base classes for particular analysis or problem type, represented by corresponding engng model. They typically declare the general services required for a specific analysis purpose - like evaluation of stiffness or mass matrices for structural analysis, or evaluation of capacity and conductivity matrices for heat transfer analysis. Usually they also provide general implementation of these services.

The **Element** is derived from a **FEMComponent**. It inherits the **FEMComponent**'s ability to keep its number and reference to the domain, it belongs to, its error and warning reporting services, methods for field extraction functions from object record (used when element reads its description from input database). Also the **FEMComponent** declares several abstract services. The most important are *initializeFrom* for object initialization from a given record, *saveContext* and *restoreContext* methods for storing and restoring object state to/from a stream, and *giveInterface* service for requesting an object interface.

The attributes declared at the **Element** class include the variables used to keep its list of nodes and sides, its material and cross section number, lists of applied body and boundary loads, list of integration rules and array storing its code numbers (see OOFEM library reference manual for details).

The following important services are declared at the element level:

- services for component management - include services for requesting element's geometrical characteristics (*giveDofManager*, *giveNode*, *giveNumberOfDofManagers*, *giveNumberOfNodes*), material (*giveMaterial*), cross section (*giveCrossSection*).
- services related to code numbers management:
 - giveLocationArray* - returns the element location array. This location array is obtained by appending code-numbers of element nodes (according to the node numbering), followed by code numbers of element sides. The ordering of DOFs for the particular node/side is specified using a node/side DOF mask, which is obtained using *giveNodeDofIDMask* or *giveSideDofIDMask* services. Please note, that this local DOF ordering must be taken into account when assembling various local characteristic vectors and matrices. Once the element location array is assembled, it is cached and reused to avoid time consuming assembly. Some engineering models may support dynamic changes of the static system (generally, of boundary conditions) during analysis, then these models use *invalidateLocationArray* function to invalidate location array after finishing time step, to force new equation numbering.
 - computeNumberOfDofs* - computes or simply returns total number of element's local DOFs. Must be implemented by particular element. The *giveDofManDofIDMask* service return DOF mask for corresponding dof manager (node or side). This mask defines the DOFs which are used by element at the given node/side. The mask influences the code number ordering for the particular node. Code numbers are ordered according to the node order and DOFs belonging to the particular node are ordered according to this mask. If element requests DOFs using a node mask which are not in the node then error is generated. This masking allows node to be shared by different elements with different DOFs in the same node/side. Element's local code numbers are extracted from the node/side using this mask. These services must be implemented (overloaded) by particular element implementations.
- services for requesting the so-called characteristic components. (*giveCharacteristicMatrix* and *giveCharacteristicVector*). The component requested is identified by parameter of type "CharType" (see cltypes.h). These are

³ When a specific services are required at element level by other components of a system (error estimator, graphics subsystem, etc.) they can be included using so-called interface concept at particular element level using multiple inheritance.

general methods for obtaining various element contributions to the global problem. These member functions have to be overloaded by derived analysis-specific classes in order to invoke the proper method according to the type of requested component.

- services related to the solution step update and termination.
These services are used to update the internal variables at element's integration points prior to reached state (*updateYourself* and *updateInternalState*). Similar service for the internal state initialization is also declared (see *initializeYourself*) and re-initialization to previous equilibrium state (see *initForNewStep*).
- services for accessing local element's unknowns from corresponding DOFs. These include methods for requesting local element vector of unknowns (*computeVectorOf*) and local element vector of prescribed unknowns (*computeVectorOfPrescribed*).
- Services for handling transformations between element local coordinate system and coordinate system used in nodes (possibly different from global coordinate system).
- miscellaneous services. Their detailed description can be found in OOFEM library reference manual (see also "element.h" (declaration) and "element.C" (implementation) files).

4.2 Analysis specific element classes

The direct child classes of the **Element** class are assumed to be base classes for particular analyses. For example, the **StructuralElement** class is the base class for all "structural" elements. It declares all necessary services required by structural analysis (for example methods computing stiffness matrices, load, strain and stress vectors etc.). This class may provide general implementations of some of these services if possible, implemented by virtual functions (they reflect specific element formulation - like computing element shape functions), which are declared, but their implementation is left on derived classes, which implement specific elements.

4.2.1 Structural Element - Example

This class is the base class for all structural elements. It is derived from the **Element** class. The basic tasks of the structural element is to compute its contributions to global equilibrium equations (mass and stiffness matrices, various load vectors (due to boundary conditions, force loading, thermal loading, etc.) and computing the corresponding strains and stresses from nodal displacements. Therefore the corresponding virtual services for computing these contributions are declared. These standard contributions can be computed by numerical integration of appropriate terms, which typically depend on element interpolation or material model, over the element volume. Therefore, it is possible to provide general implementations of these services, provided that the corresponding methods for computing interpolation dependent terms and material terms are implemented, and corresponding integration rules are initialized. This concept will be demonstrated on service computing stiffness matrix. Since element stiffness matrix contributes to the global equilibrium, the stiffness will be requested using *giveCharacteristicMatrix* service. The implementation of the services of the structural element takes into account only material nonlinearity, the geometrical nonlinearity is not taken into account (regarding geometrical nonlinearity, see **NLStructuralElement** class). The implementation of this service is following:

```
void
StructuralElement :: giveCharacteristicMatrix (FloatMatrix& answer,
                                             CharType mtrx, TimeStep *tStep)
//
// returns characteristic matrix of receiver according to mtrx
//
{
    if (mtrx == TangentStiffnessMatrix)
```

```

    this -> computeStiffnessMatrix(answer, TangentStiffness, tStep);
else if (mtrx == SecantStiffnessMatrix)
    this -> computeStiffnessMatrix(answer, SecantStiffness, tStep);
else if (mtrx == MassMatrix)
    this -> computeMassMatrix(answer, tStep);
else if
    ....
}

```

The first parameter is the matrix object to be computed, the `mtrx` parameter determines the type of contribution and the last parameter, time step, represents time. The element stiffness matrix can be evaluated using

$$\mathbf{K} = \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV,$$

where \mathbf{B} is the so-called geometrical matrix, containing derivatives of shape functions and \mathbf{D} is the material stiffness matrix. If \mathbf{D} is symmetric (which is usually the case) then element stiffness is symmetric, too. The numerical integration is used to evaluate this integral. For numerical integration, the **IntegrationRule** class is used. The integration rules for a specific element are created during element initialization and are stored in `integrationRulesArray` attribute, inherited from the **Element** class. In order to implement the stiffness evaluation, the methods for computing geometrical matrix and material stiffness matrix are declared (as virtual), but not implemented. They have to be implemented by specific elements, because they “know” their interpolation and the corresponding material mode. The implementation of `computeStiffnessMatrix` is following

```

void
StructuralElement :: computeStiffnessMatrix (FloatMatrix& answer,
                                             MatResponseMode rMode, TimeStep* tStep)
// Computes numerically the stiffness matrix of the receiver.
{
    int j;
    double dV ;
    FloatMatrix d, bj, dbj;
    GaussPoint *gp ;
    IntegrationRule* iRule;

    // give reference to integration rule
    iRule = integrationRulesArray[giveDefaultIntegrationRule()];

    // loop over integration points
    for (j=0 ; j < iRule->getNumberOfIntegrationPoints() ; j++) {
        gp = iRule->getIntegrationPoint(j) ;
        // compute geometrical matrix of particular element
        this -> computeBmatrixAt(gp, bj) ;
        //compute material stiffness
        this -> computeConstitutiveMatrixAt(d, rMode, gp, tStep);
        // compute jacobian
        dV = this -> computeVolumeAround(gp) ;
        // evaluate stiffness
        dbj.beProductOf (d, bj) ;
        answer.plusProductSymmUpper(bj,dbj,dV) ;
    }
}

```



```

    }

    answer.symmetrized() ;
    // transform into global coordinate system if necessary
    if (this->updateRotationMatrix())
        answer.rotatedWith(*this->rotationMatrix) ;
    return  ;
}

```

Inside the integration loop, only the upper half of the element stiffness is computed in element local coordinate system. Then, the lower part of the stiffness is initialized from the upper part (answer.symmetrized()) and transformation from element local coordinate system to global coordinate system is done. The transformation matrix is cached in the rotationMatrix attribute declared in the **StructuraleElement** class. The *updateRotationMatrix* service updates this matrix (if necessary) and returns pointer to this matrix. The zero value indicates that no transformation is necessary. The other element contributions can be computed using similar procedures. In general, different integration rules can be used for evaluation of different element contributions. For example, the support for the reduced integration of some terms of the stiffness matrix can be implemented - see the implementation of *computeStiffnessMatrix* in “structuralelement.C”.

The element strain and stress vectors at a given integration point are computed using *computeStrainVector* and *computeStressVector* services. The element strain vector can be evaluated using $\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{u}$, where \mathbf{B} is the geometrical matrix and \mathbf{u} is element local displacement vector. The stress evaluation is rather simple, since the stress evaluation from a given strain increment and actual state (kept within the integration point) is done at the cross section description level (integration over cross section volume) and material model level (stress evaluation).

```

void
StructuralElement :: computeStrainVector (FloatArray& answer,
                                           GaussPoint* gp, TimeStep* stepN)
// Computes the vector containing the strains
// at the Gauss point gp of the receiver,
// at time step stepN. The nature of these strains depends
// on the element's type.
{
    FloatMatrix b;
    FloatArray u ;

    this -> computeBmatrixAt(gp, b) ;
    // compute vector of element's unknowns
    this -> computeVectorOf(DisplacementVector,
                           UnknownMode_Total,stepN,u) ;
    // transform global unknowns into element local c.s.
    if (this->updateRotationMatrix())
        u.rotatedWith(this->rotationMatrix,'n') ;
    answer.beProductOf (b, u) ;

    return ;
}

```

[illegible]

```

// Computes the vector containing the stresses
// at the Gauss point gp of the receiver, at time step stepN.
// The nature of these stresses depends on the element's type.
{

    FloatArray Epsilon ;
    StructuralCrossSection* cs = (StructuralCrossSection*)
                                this->giveCrossSection();
    Material *mat = this->giveMaterial();

    this->computeStrainVector (Epsilon, gp, stepN) ;
    // ask cross section model for real stresses
    // for given strain increment
    cs -> giveRealStresses (answer, ReducedForm, gp, Epsilon, stepN);

    return ;
}

```

The **StructuralElement** class overloads also services for context storing/restoring, calling corresponding services for all integration rules (and thus on all integration points belonging to an element).

For further reference see OOFEM library reference manual and files “structuralelement.h” and “structuralelement.C”.

4.2.2 Isoparametric Truss 2d element

```

class Truss2d : public StructuralElement
{
/*
This class implements a two-node truss bar element
for two-dimensional analysis.
A truss bar element is characterized by its 'length'
and its 'pitch'. The pitch is the angle in radians
between the X-axis and the axis of the element
(oriented node1 to node2).
*/

protected :
    double      length ;
    double      pitch ;

public :
    Truss2d (int, Domain*) ;    // constructor
    ~Truss2d () {}             // destructor

    // mass matrix computations
    void computeLumpedMassMatrix (FloatMatrix& answer,
                                   TimeStep* tStep) ;

    // general mass service overloaded
    void computeMassMatrix (FloatMatrix& answer, TimeStep* tStep)
        {computeLumpedMassMatrix(answer, tStep);}
}

```

```

        // DOF management
virtual int      computeNumberOfDofs (EquationID ut) {return 4;}
virtual void     giveDofManDofIDMask (int inode, EquationID, IntArray& ) const;

        double computeVolumeAround (GaussPoint*) ;

//
// definition & identification
//
char* giveClassName (char* s) const
    { return strcpy(s,"Truss2d") ;}
classType giveClassID () const { return Truss2dClass; }

IRResultType initializeFrom (InputRecord* ir);

protected:
    // computes geometrical matrix
    void computeBmatrixAt (GaussPoint*, FloatMatrix&,
                          int=1, int=ALL_STRAINS) ;
    // computes interpolation matrix
    void computeNmatrixAt (GaussPoint*, FloatMatrix&) ;
    // initialize element's integration rules
    void computeGaussPoints () ;
    // transformation from global->local c.s.
    int  computeGtoLRotationMatrix (FloatMatrix&);

    double giveLength () ;
    double givePitch () ;
} ;

```

This is a minimal-functionality implementation. It does not include, for example, any support for element loading or material nonlinearity. The implementation starts with standard constructor and destructor.

```

Truss2d :: Truss2d (int n, Domain* aDomain) :
    StructuralElement (n,aDomain)
// Constructor.
{
    numberOfDofMans      = 2 ;
    rotationMatrix       = NULL ;
    length              = 0. ;
    pitch               = 10. ;    // a dummy value
}

```

Next, the services for computing interpolation and geometrical matrices follow. The implementation of the *computeNmatrixAt* is not necessary for the current purpose, since the mass matrix computation is overloaded and does not

require this service, but it is added for completeness. The both methods compute response at a given integration point, which is passed as a parameter. The *computeBmatrixAt* has two additional parameters which determine the range of strain components for which response is assembled. This has something to do with support for reduced/selective integration and is not important in presented simple case.

Recently, the interpolation classes have been added, that can significantly facilitate the element implementation. They provide shape functions, their derivatives, transformation jacobians out of the box.

```
void
Truss2d :: computeNmatrixAt (GaussPoint* aGaussPoint,
                             FloatMatrix& answer)
// Returns the displacement interpolation matrix {N}
// of the receiver, evaluated at aGaussPoint.
{
    double      ksi,n1,n2 ;

    ksi = aGaussPoint -> giveCoordinate(1) ;
    n1  = (1. - ksi) * 0.5 ;
    n2  = (1. + ksi) * 0.5 ;

    answer.resize (2,4);
    answer.zero();

    answer.at(1,1) = n1 ;
    answer.at(1,3) = n2 ;
    answer.at(2,2) = n1 ;
    answer.at(2,4) = n2 ;

    return ;
}

void
Truss2d :: computeBmatrixAt (GaussPoint* aGaussPoint,
                             FloatMatrix& answer, int li, int ui)
//
// Returns linear part of geometrical
// equations of the receiver at gp.
// Returns the linear part of the B matrix
//
{
    double coeff,l;

    answer.resize(1,4);
    l = this->giveLength();
    coeff = 1.0/l;

    answer.at(1,1) =-coeff;
    answer.at(1,2) = 0.0;
    answer.at(1,3) = coeff;
    answer.at(1,4) = 0.0;
```

```

    return;
}

```

The following two functions compute the basic geometric characteristics of a bar element - its length and pitch, defined as the angle between global x-axis and the local element x-axis (oriented from node1 to node2).

```

double Truss2d :: giveLength ()
    // Returns the length of the receiver.
{
    double dx,dz ;
    Node    *nodeA,*nodeB ;

    if (length == 0.) {
        nodeA = this->giveNode(1) ;
        nodeB = this->giveNode(2) ;
        dx    = nodeB->giveCoordinate(1)-nodeA->giveCoordinate(1);
        dz    = nodeB->giveCoordinate(3)-nodeA->giveCoordinate(3);
        length= sqrt(dx*dx + dz*dz) ;}

    return length ;
}

double Truss2d :: givePitch ()
    // Returns the pitch of the receiver.
{
    double xA,xB,zA,zB ;
    Node    *nodeA,*nodeB ;

    if (pitch == 10.) { // 10. : dummy initialization value
        nodeA = this -> giveNode(1) ;
        nodeB = this -> giveNode(2) ;
        xA    = nodeA->giveCoordinate(1) ;
        xB    = nodeB->giveCoordinate(1) ;
        zA    = nodeA->giveCoordinate(3) ;
        zB    = nodeB->giveCoordinate(3) ;
        pitch = atan2(zB-zA,xB-xA) ;}

    return pitch ;
}

```

When an element is created (by the **Domain** class), the default constructor is called. To initialize the element, according to its record in the input database, the *initializeFrom* is immediately called after element creation. The element implementation should first call the parent implementation to ensure that attributes declared at parent level are initialized properly. Then the element has to initialize attributes declared by itself and also to set up its integration rules. In our example, special method *computeGaussPoints* is called to initialize integration rules. In this case, only

one integration rule is created. It is of type **GaussIntegrationRule**, indicating that the Gaussian integration is used. Once integration rule is created, its integration points are created to represent line integral, with 1 integration point. Integration points will be associated to element under consideration and will have 1D material mode (which determines the type of material model response).

```

IRResultType
Truss2d :: initializeFrom (InputRecord* ir)
{
    this->NLStructuralElement :: initializeFrom (ir);
    this -> computeGaussPoints();
    return IRRT_OK;
}

void Truss2d :: computeGaussPoints ()
    // Sets up the array of Gauss Points of the receiver.
{
    numberOfIntegrationRules = 1 ;
    integrationRulesArray = new IntegrationRule*;
    integrationRulesArray[0] = new GaussIntegrationRule (1,domain, 1, 2);
    integrationRulesArray[0]->
        setUpIntegrationPoints (_Line, 1, this, _1dMat);
}

double Truss2d :: computeVolumeAround (GaussPoint* aGaussPoint)
    // Returns the volume corresponding to given integration point.
{
    double weight = aGaussPoint -> giveWeight() ;
    return 0.5 * this->giveLength() * weight *
        this->giveCrossSection()->give('A');
}

void
Truss2d :: computeLumpedMassMatrix (FloatMatrix& answer, TimeStep* tStep)
    // Returns the lumped mass matrix of the receiver. This expression is
    // valid in both local and global axes.
{
    Material* mat ;
    double halfMass ;

    mat = this -> giveMaterial() ;
    halfMass = mat->give('d') *
        this->giveCrossSection()->give('A') *
        this->giveLength() / 2.;

    answer.resize (4,4) ; answer.zero();
    answer . at(1,1) = halfMass ;
    answer . at(2,2) = halfMass ;
    answer . at(3,3) = halfMass ;
}

```

```

    answer . at(4,4) = halfMass ;

    if (this->updateRotationMatrix())
        answer.rotatedWith(*this->rotationMatrix) ;
    return ;
}

```

The following service computes the part of the element transformation matrix, corresponding to transformation between global and element local coordinate systems. This method is called from the general *updateRotationMatrix* service, implemented at the **StructuralElement** level, which computes the element transformation matrix, taking into account further transformations (nodal coordinate system, for example). The default implementation of *updateRotationMatrix* computes the element transformation matrix only once and stores it in rotationMatrix attribute. Since this service is virtual, particular elements can overload this service to take into account continuously changing local coordinate system (corotational elements in nonlinear analysis).

```

int
Truss2d :: computeGtoLRotationMatrix (FloatMatrix& rotationMatrix)
// computes the rotation matrix of the receiver.
// r(local) = T * r(global)
{
    double sine,cosine ;

    sine          = sin (this->givePitch()) ;
    cosine        = cos (pitch) ;

    rotationMatrix.resize(4,4);
    rotationMatrix . at(1,1) = cosine ;
    rotationMatrix . at(1,2) = sine   ;
    rotationMatrix . at(2,1) = -sine  ;
    rotationMatrix . at(2,2) = cosine ;
    rotationMatrix . at(3,3) = cosine ;
    rotationMatrix . at(3,4) = sine   ;
    rotationMatrix . at(4,3) = -sine  ;
    rotationMatrix . at(4,4) = cosine ;

    return 1 ;
}

void
Truss2d :: giveDofManDofIDMask (int inode, EquationID, IntArray& answer) const {
// returns DofId mask array for inode element node.
// DofId mask array determines the dof ordering requested from node.
// DofId mask array contains the DofID constants (defined in cltypes.h)
// describing physical meaning of particular DOFs.
//IntArray* answer = new IntArray (2);
    answer.resize (2);

    answer.at(1) = D_u;
    answer.at(2) = D_w;
}

```

```

    return ;
}

```

4.3 Material model interface

The base class for all material models is the class **Material**, derived from the **FEMComponent**. It declares analysis independent part of the material interface. The analysis specific part of the interface should be added by derived classes, which represent the base classes for specific problem. The typical example is the **StructuralMaterial** class which declares all the services necessary for the structural analysis. Services declared or implemented at **Material** level include

- Material status related services. The material model has to be able to create instance of corresponding material status, where the history variables are stored. This is done by invoking *CreateStatus*. The status corresponding to a given integration point can be requested using the *giveStatus* service.
- Services for integration point update and initialization. There are generally two sets of history variables kept in corresponding material statuses for each integration point. One set is referring to previous equilibrated state, the second one to the actual state during the solution (more precisely to the achieved local-equilibrium state), which may not correspond to the global equilibrium state. The methods are provided to update the actual state as equilibrated (*updateYourself* service) and for initialization of actual state to previous equilibrium (*initTempStatus*). The implementation of these services simply extract the corresponding status of a given integration point and calls the corresponding service of material status.
- Services for testing the material model capabilities (*testMaterialExtension*, *hasMaterialModeCapability*, and *hasNonLinearBehaviour*).
- Services for requesting internal variables and properties.

4.4 Analysis specific material model classes

The direct derived classes of the **Material** class are supposed to declare the analysis specific part of the material model interface, which is required (and assumed) by the corresponding element class (**StructuralElement**) and cross section class (**StructuralCrossSection**).

To store all necessary history variables of the material model, so called status concept is adopted. The material status can be thought as container of all necessary history variables. Usually two kinds of these variables are stored. The temporary ones refer to the actual state of an integration point, but do not necessary correspond to the global equilibrium. These are changing during global equilibrium search iteration. The non-temporary variables are related to the previously converged state. For each material model, the corresponding status has to be defined, and the unique instance for each integration point has to be created and associated with it. The integration point provides the services for accessing corresponding status. All material statuses, related to particular material models, have to be derived from the base **MaterialStatus** class. This class declares the basic status interface. The two most important services are: *initTempStatus* intended to initialize the temporary internal variables according to variables related to previously reached equilibrium state and *updateYourself* designed to update the equilibrium-like history variables according to temporary variables, when the new global equilibrium has been reached. The derived classes should also define methods for accessing the corresponding history variables.

4.4.1 Structural Material class - Example

The structural (or mechanical) constitutive model should generally support several so-called material models, corresponding to various modeling assumptions (plane-stress, plane-strain, or 1D-behavior, for example). The concept of multiple material modes is generally supported by **Material**, which provides the services for testing the model capabilities. It is generally assumed, that results obtained from constitutive model services are formed according to material mode. This mode is attribute of each integration point, which is compulsory parameter of all material services. For computational convenience, the so-called full and reduced formats of stress/strains vectors are introduced, corresponding to material modes. The full format includes all components, even if they are zero due to stress/strain mode nature. In the reduced format, only generally nonzero components are stored. (Full format should be used only if absolutely necessary, to avoid wasting of space. For example, it is used by output routines to print results in general form). Methods for converting vectors between full and reduced format are provided. If possible, all computations should be performed in reduced space.

The convention used to construct reduced strain/stress vectors is generally as follows. If in a particular mode a particular stress component is zero, the corresponding strain is not computed and not stored in reduced vector, and in full vector there is zero value on corresponding position. On the other hand, if zero strain component is imposed, then this condition must be taken into account in geometrical relations (at element level), and corresponding components are included in stress/strain reduced vectors.

Generally, the following major tasks are declared by **StructuralMaterial** or inherited from **Material** class:

- Computing the real stress vector (tensor) at an integration point for a given strain increment and updating its temporary state corresponding to the local equilibrium, but not necessarily to the global equilibrium (see *giveRealStressVector*). The parameters include the total strain vector and the corresponding integration point. The total strain is defined as strain computed directly from the displacement field at a given time. The stress independent parts (temperature, eigen strains) should be subtracted and the corresponding load-history variables (stored in the corresponding status) can be used. The temporary history variables in the status should be updated according to the newly reached state. The temporary history variables are moved into equilibrium history variables just after the global structure equilibrium has been reached by the iteration process.
- Updating the integration point state (final state), when the global equilibrium has been reached.
- Returning material stiffness and/or flexibility matrices for a given material mode. The general methods computing the response for the specific material mode are provided, based on converting 3D stiffness or compliance matrix to that corresponding to the specific material mode. But, if it is possible to compute stiffness or compliance matrix directly for the specific mode, then these general methods should be overloaded.
- Storing/restoring integration point state to/from a stream.
- Requesting internal variables, their types and properties (*giveIPValue*, *giveIPValueSize*, *giveIPValueType*, and *giveIntVarCompFullIndx* services).
- Returning material properties.

Structural material services should not be called directly by elements. Instead, the elements should pass their requests to the corresponding cross section model, that performs all necessary integration over its volume and invokes corresponding material model services.

The **StructuralMaterial** class comes with definition of associated material status - **StructuralMaterialStatus**. This is only an abstract class. For every instance of **StructuralMaterial** class there should be a specialized derived class, which maintains all history variables. It only adds attributes common to all “structural analysis” material models - the strain and stress vectors (both the temporary-like, corresponding to the local equilibrium and non-temporary ones, corresponding to the global equilibrium). The corresponding services for accessing, setting, initializing, and updating these attributes are provided.

4.4.2 Isotropic Damage Model

In this section, the implementation of an isotropic damage model will be described. To cover the various models based on isotropic damage concept, a base class **IsotropicDamageMaterial** is defined first, declaring the necessary services and providing the implementation of them, which are general. The derived classes then only implement a particular damage-evolution law.

The isotropic damage models are based on the simplifying assumption that the stiffness degradation is isotropic, i.e., stiffness moduli corresponding to different directions decrease proportionally and independently of direction of loading. Consequently, the damaged stiffness matrix is expressed as

$$\mathbf{D} = (1 - \omega)\mathbf{D}_e,$$

where \mathbf{D}_e is elastic stiffness matrix of the undamaged material and ω is the damage parameter. Initially, ω is set to zero, representing the virgin undamaged material, and the response is linear-elastic. As the material undergoes the deformation, the initiation and propagation of microdefects decreases the stiffness, which is represented by the growth of the damage parameter ω . For $\omega = 1$, the stiffness completely disappears.

In the present context, the \mathbf{D} matrix represents the secant stiffness that relates the total strain to the total stress

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} = (1 - \omega)\mathbf{D}_e\boldsymbol{\varepsilon}.$$

Similarly to the theory of plasticity, a loading function f is introduced. In the damage theory, it is natural to work in the strain space and therefore the loading function is depending on the strain and on an additional parameter κ , describing the evolution of the damage. Physically, κ is a scalar measure of the largest strain level ever reached. The loading function usually has the form

$$f(\boldsymbol{\varepsilon}, \kappa) = \tilde{\varepsilon}(\boldsymbol{\varepsilon}) - \kappa,$$

where $\tilde{\varepsilon}$ is the equivalent strain, i.e., the scalar measure of the strain level. Damage can grow only if current state reaches the boundary of elastic domain ($f = 0$). This is expressed by the following loading/unloading conditions

$$f \leq 0, \quad \dot{\kappa} \geq 0, \quad \dot{\kappa}f = 0.$$

It remains to link the variable κ to the damage parameter ω . As both κ and ω grow monotonically, it is convenient to postulate an explicit evolution law

$$\omega = g(\kappa).$$

The important advantage of this explicit formulation is that the stress corresponding to the given strain can be evaluated directly, without the need to solve the nonlinear system of equations. For the given strain, the corresponding stress is computed simply by evaluating the current equivalent strain, updating the maximum previously reached equivalent strain value κ and the damage parameter and reducing the effective stress according to $\boldsymbol{\sigma} = (1 - \omega)\mathbf{D}_e\boldsymbol{\varepsilon}$. This general framework for computing stresses and stiffness matrix is common for all material models of this type. Therefore, it is natural to introduce the base class for all isotropic-based damage models which provides the general implementation for the stress and stiffness matrix evaluation algorithms. The particular models then only provide their equivalent strain and damage evolution law definitions. The base class only declares the virtual services for computing equivalent strain and corresponding damage. The implementation of common services uses these virtual functions, but they are only declared at **IsotropicDamageMaterial** class level and have to be implemented by the derived classes.

Together with the material model, the corresponding status has to be defined, containing all necessary history variables. For the isotropic-based damage models, the only history variable is the value of the largest strain level ever reached (κ). In addition, the corresponding damage level ω will be stored. This is not necessary because damage can be always computed from corresponding κ . The **IsotropicDamageMaterialStatus** class is derived from **StructuralMaterialStatus** class. The base class represents the base material status class for all structural statuses. At **StructuralMaterialStatus** level, the attributes common to all “structural analysis” material models - the strain

and stress vectors (both the temporary and non-temporary) are introduced. The corresponding services for accessing, setting, initializing, and updating these attributes are provided. Therefore, only the κ and ω parameters are introduced (both the temporary and non-temporary). The corresponding services for manipulating these attributes are added and services for context initialization, update, and store/restore operations are overloaded, to handle the history parameters properly.

```

class IsotropicDamageMaterialStatus : public StructuralMaterialStatus
{
protected:
    /// scalar measure of the largest strain level ever reached in material
    double kappa;
    /// non-equilibrated scalar measure of the largest strain level
    double tempKappa;
    /// damage level of material
    double damage;
    /// non-equilibrated damage level of material
    double tempDamage;

public:
    /// Constructor
    IsotropicDamageMaterialStatus (int n, Domain*d, GaussPoint* g) ;
    /// Destructor
    ~IsotropicDamageMaterialStatus ();

    /// Prints the receiver state to stream
    void printOutputAt (FILE *file, TimeStep* tStep) ;

    /// Returns the last equilibrated scalar measure
    /// of the largest strain level
    double giveKappa () {return kappa;}
    /// Returns the temp. scalar measure of the
    /// largest strain level
    double giveTempKappa () {return tempKappa;}
    /// Sets the temp scalar measure of the largest
    /// strain level to given value
    void setTempKappa (double newKappa) { tempKappa = newKappa;}
    /// Returns the last equilibrated damage level
    double giveDamage () {return damage;}
    /// Returns the temp. damage level
    double giveTempDamage () {return tempDamage;}
    /// Sets the temp damage level to given value
    void setTempDamage (double newDamage) { tempDamage = newDamage;}

    // definition
    char* giveClassName (char* s) const
    { return strcpy(s,"IsotropicDamageMaterialModelStatus") ;}
    classType giveClassID () const
    { return IsotropicDamageMaterialStatusClass; }

    /**
     * Initializes the temporary internal variables,

```

```

        describing the current state according to
        previously reached equilibrium internal variables.
    */
    virtual void initTempStatus ();
    /**
        Update equilibrium history variables
        according to temp-variables.
        Invoked, after new equilibrium state has been reached.
    */
    virtual void updateYourself(TimeStep*);

    // saves current context(state) into stream
    /**
        Stores context of receiver into given stream.
        Only non-temp internal history variables are stored.
        @param stream stream where to write data
        @param obj pointer to integration point, which invokes this method
        @return nonzero if o.k.
    */
    contextIOResultType    saveContext (FILE* stream, void *obj = NULL);
    /**
        Restores context of receiver from given stream.
        @param stream stream where to read data
        @param obj pointer to integration point, which invokes this method
        @return nonzero if o.k.
    */
    contextIOResultType    restoreContext(FILE* stream, void *obj = NULL);

};

```

The base **IsotropicDamageMaterial** class is derived from the **StructuralMaterial** class.

```

/**
    Base class representing general isotropic damage model.
    It is based on isotropic damage concept,
    assuming that damage evolution law
    is postulated in explicit form,
    relating damage parameter (omega) to scalar measure
    of the largest strain level ever reached in material (kappa).
*/
class IsotropicDamageMaterial : public StructuralMaterial
{
protected :
    /// Coefficient of thermal dilatation
    double tempDillatCoeff;

    /// Reference to bulk (undamaged) material
    LinearElasticMaterial *linearElasticMaterial;

    /**
        variable controlling type of loading/unloading law,
        default set to idm_strainLevel
    */

```

[illegible]

```

                                GaussPoint* gp,
                                TimeStep* atTime);

/**
 Computes the real stress vector for given strain
 increment and integration point.
 Temp vars are updated accordingly
 @param answer contains result
 @param form material response form
 @param gp integration point
 @param reducedStrain strain vector in reduced form
 @param tStep current time step (most models are able to
 respond only when tStep is current time step)
 */
void giveRealStressVector (FloatArray& answer, MatResponseForm form,
                           GaussPoint* gp, const FloatArray &reducedStrain,
                           TimeStep* tStep);

/**
 Returns the integration point corresponding
 value in Reduced form.
 @param answer contain corresponding ip value,
 zero sized if not available
 @param aGaussPoint integration point
 @param type determines the type of internal variable
 @returns nonzero if ok, zero if var not supported
 */
virtual int giveIPValue (FloatArray& answer,
                        GaussPoint* aGaussPoint,
                        InternalStateType type,
                        TimeStep* atTime) ;

/**
 Returns the mask of reduced indexes of
 Internal Variable component .
 @param answer mask of Full VectorSize, with components
 being the indexes to reduced form vectors.
 @param type determines the internal variable requested
 @returns nonzero if ok or error is generated for unknown mat mode.
 */
virtual int giveIntVarCompFullIndx (IntArray& answer,
                                    InternalStateType type,
                                    MaterialMode mmode);

/**
 Returns the type of internal variable (scalar, vector, ...)
 @param type determines the type of internal variable
 @returns type of internal variable
 */
virtual
InternalStateValueType giveIPValueType (InternalStateType type);

```

```

/**
    Returns the corresponding integration point
    value size in Reduced form.
    @param type determines the type of internal variable
    @returns var size, zero if var not supported
*/
virtual int giveIPValueSize (InternalStateType type,
                             GaussPoint* aGaussPoint) ;

/**
    Returns a vector of coefficients of thermal dilatation in direction
    of each material principal (local) axis.
    @param answer vector of thermal dilatation coefficients
    @param gp integration point
    @param tStep time step (most models are able to
    respond only when atTime is current time step)
*/
virtual void giveThermalDilatationVector (FloatArray& answer,
                                           GaussPoint*, TimeStep*) ;

/**
    Computes the equivalent strain measure from
    given strain vector (full form).
    @param kappa return param, containing the
    corresponding equivalent strain
    @param strain total strain vector in full form
    @param gp integration point
    @param atTime timestep
*/
virtual void computeEquivalentStrain (double& kappa,
                                      const FloatArray& strain,
                                      GaussPoint* gp,
                                      TimeStep* atTime) = 0;

/**
    computes the value of damage parameter omega,
    based on given value of equivalent strain
    @param omega contains result
    @param kappa equivalent strain measure
*/
virtual void computeDamageParam (double& omega, double kappa,
                                 const FloatArray& strain,
                                 GaussPoint* gp) =0;

/**
    Instantiates the receiver from input record
*/
IRResultType initializeFrom (InputRecord* ir);

protected:

/**

```

```

Abstract service allowing to perform some
initialization, when damage first appear
@param kappa scalar measure of strain level
@param totalStrainVector current total strain vector
@param gp integration point
*/
virtual void initDamaged (double kappa,
                          FloatArray& totalStrainVector,
                          GaussPoint* gp) {}

/// Creates corresponding material status
MaterialStatus* CreateStatus (GaussPoint *gp)
{return new IsotropicDamageMaterialStatus (1,domain,gp);}

// Overloaded to use specialized versions of these
// services possibly implemented by linearElastic member
/**
Method for computing plane stress stiffness matrix of receiver.
Default implementation overloaded to use direct implementation of
corresponding service at bulk material model level.
@param answer stiffness matrix
@param form material response form
@param mode material response mode
@param gp integration point, which load history is used
@param atTime time step (most models are able to
respond only when atTime is current time step)
*/
void givePlaneStressStiffMtrx (FloatMatrix& answer,
                              MatResponseForm form,
                              MatResponseMode,
                              GaussPoint* gp,
                              TimeStep* atTime);

/**
Method for computing plane strain stiffness matrix of receiver.
Default implementation overloaded to use direct implementation of
corresponding service at bulk material model level.
@param answer stiffness matrix
@param form material response form
@param mode material response mode
@param gp integration point, which load history is used
@param atTime time step (most models are able to
respond only when atTime is current time step)
*/
void givePlaneStrainStiffMtrx (FloatMatrix& answer,
                              MatResponseForm form,
                              MatResponseMode,
                              GaussPoint* gp,
                              TimeStep* atTime);

/**

```



```

    Method for computing 1d stiffness matrix of receiver.
    Default implementation overloaded to use direct implementation of
    corresponding service at bulk material model level.
    @param answer stiffness matrix
    @param form material response form
    @param mode material response mode
    @param gp integration point, which load history is used
    @param atTime time step (most models are able to
        respond only when atTime is current time step)
    */
void give1dStressStiffMtrx (FloatMatrix& answer,
                           MatResponseForm form,
                           MatResponseMode,
                           GaussPoint* gp,
                           TimeStep* atTime);
} ;

#include "isodamagemodel.h"
#include "gausspnt.h"
#include "flotmtrx.h"
#include "flotarray.h"
#include "structuralcrosssection.h"
#include "mathfem.h"

IsotropicDamageMaterial :: IsotropicDamageMaterial (int n, Domain *d)
: StructuralMaterial (n,d)
//
// constructor
//
{
    linearElasticMaterial = NULL;
    llcriteria = idm_strainLevelCR;
}

IsotropicDamageMaterial :: ~IsotropicDamageMaterial ()
//
// destructor
//
{
    delete linearElasticMaterial;
}

int
IsotropicDamageMaterial :: hasMaterialModeCapability (MaterialMode mode)
//
// returns whether receiver supports given mode
//
{
    if ((mode == _3dMat) || (mode == _PlaneStress) ||
        (mode == _PlaneStrain) || (mode == _1dMat)) return 1;
    return 0;
}

```

```
}

```

Next, the implementation for core services is presented. They compute material stiffness matrices for various material modes and compute the stress vector. The later service computes the real stress corresponding to the previous history and the given strain vector. It computes a new locally consistent (equilibrated) state, which is stored in temporary variables of the corresponding status (attribute of the integration point).

```
void
IsotropicDamageMaterial :: give3dMaterialStiffnessMatrix (FloatMatrix& answer,
                                                         MatResponseForm form,
                                                         MatResponseMode mode,
                                                         GaussPoint* gp,
                                                         TimeStep* atTime)

//
// computes full constitutive matrix for case of gp stress-strain state.
//
{
    IsotropicDamageMaterialStatus *status =
        (IsotropicDamageMaterialStatus*) this -> giveStatus (gp);
    double om = status->giveTempDamage();
    om = min (om, 0.999999);

    this->giveLinearElasticMaterial()->
        give3dMaterialStiffnessMatrix (answer, form, mode, gp, atTime);
    answer.times (1.0-om);

    return ;
}

```

```
void
IsotropicDamageMaterial :: giveRealStressVector (FloatArray& answer,
                                                MatResponseForm form,
                                                GaussPoint* gp,
                                                const FloatArray& totalStrain,
                                                TimeStep* atTime)

//
// returns real stress vector in 3d stress space of receiver according to
// previous load history and given strain.
// After the new local point equilibrium is reached all the
// temp history variables are updated to new local equilibrium state.
//
//
{
    IsotropicDamageMaterialStatus *status =
        (IsotropicDamageMaterialStatus*) this -> giveStatus (gp);

    LinearElasticMaterial* lmat = this->giveLinearElasticMaterial();
    FloatArray strainVector, reducedTotalStrainVector;
    FloatMatrix de;
    double f, equivStrain, tempKappa, omega;

    this->initGpForNewStep(gp);
}

```

```

// subtract stress independent part
// note: eigenStrains (temperature) is not contained in mechanical
// strain stored in gp therefore it is necessary to subtract
// always the total eigen strain value
this->giveStressDependentPartOfStrainVector(reducedTotalStrainVector,
                                           gp, totalStrain, atTime, TotalMode);

// compute equivalent strain
this->computeEquivalentStrain (equivStrain, reducedTotalStrainVector, gp, atTime);

if (llcriteria == idm_strainLevelCR) {
  // compute value of loading function if strainLevel crit apply
  f = equivStrain - status->giveKappa();

  if (f <= 0.0) {
    // damage do not grow
    tempKappa = status->giveKappa();
    omega     = status->giveDamage();
  } else {
    // damage grow
    tempKappa = equivStrain;
    this->initDamaged (tempKappa, reducedTotalStrainVector, gp);
    // evaluate damage parameter
    this->computeDamageParam (omega, tempKappa, reducedTotalStrainVector, gp);
  }
} else if (llcriteria == idm_damageLevelCR) {

  // evaluate damage parameter first
  tempKappa = equivStrain;
  this->initDamaged (tempKappa, reducedTotalStrainVector, gp);
  this->computeDamageParam (omega, tempKappa, reducedTotalStrainVector, gp);
  if (omega < status->giveDamage()) {
    // unloading takes place
    omega = status->giveDamage();
  }
} else _error ("giveRealStressVector: unsupported loading/unloading criteria");

lmat->giveCharacteristicMatrix(de, ReducedForm, SecantStiffness, gp, atTime);
de.times(1.0-omega);
answer.beProductOf (de, reducedTotalStrainVector);

// update gp
status-> letTempStrainVectorBe (totalStrain);
status-> letTempStressVectorBe (answer);
status-> setTempKappa (tempKappa);
status-> setTempDamage(omega);
return ;
}

/*

```

The following routines are not necessary, since the base StructuralMaterial class implements general algorithm how to derive the stiffness and compliance matrices for specific modes from full 3d stiffness/compliance.

However, these general services are not efficient, since they require matrix inversion. The better and efficient way is to overload these service and provide efficient implementation, if available.

 $\ast/$

void

```
IsotropicDamageMaterial::givePlaneStressStiffMtrx (FloatMatrix& answer,
MatResponseForm form,
MatResponseMode mode,
GaussPoint* gp,
TimeStep* atTime)
```

```

{
    IsotropicDamageMaterialStatus *status =
        (IsotropicDamageMaterialStatus*) this -> giveStatus (gp);
    double om = status->giveTempDamage();
    om = min (om, 0.999999);
}

```

```
this->giveLinearElasticMaterial()->
    giveCharacteristicMatrix (answer, form, mode, gp, atTime);
answer.times (1.0-om);
```

```
return ;
```

}

```
void
```

```
IsotropicDamageMaterial::givePlaneStrainStiffMtrx (FloatMatrix& answer,
MatResponseForm form,
MatResponseMode mode,
GaussPoint* gp,
TimeStep* atTime)
```

```
{
  IsotropicDamageMaterialStatus *status =
    (IsotropicDamageMaterialStatus*) this -> giveStatus (gp);
  double om = status->giveTempDamage();
  om = min (om, 0.9999999);
}
```

```
this->giveLinearElasticMaterial()->
    giveCharacteristicMatrix (answer, form, mode, gp, atTime);
answer.times (1.0-om);
```

```
return ;
```

}

```
void
```

[illegible]

```

MatResponseMode mode,
GaussPoint* gp,
TimeStep* atTime)
{
    IsotropicDamageMaterialStatus* status =
        (IsotropicDamageMaterialStatus*) this -> giveStatus (gp);
    double om = status->giveTempDamage();
    om = min (om, 0.999999);

    this->giveLinearElasticMaterial()->
        giveCharacteristicMatrix (answer, form, mode, gp, atTime);
    answer.times (1.0-om);

    return ;
}

int
IsotropicDamageMaterial::giveIPValue (FloatArray& answer,
                                       GaussPoint* aGaussPoint,
                                       InternalStateType type,
                                       TimeStep* atTime)
{
    IsotropicDamageMaterialStatus* status =
        (IsotropicDamageMaterialStatus*) this -> giveStatus (aGaussPoint);
    if (type == IST_DamageTensor) {
        answer.resize(1);
        answer.at(1) = status->giveDamage();
        return 1;
    } else if (type == IST_DamageTensorTemp) {
        answer.resize(1);
        answer.at(1) = status->giveTempDamage();
        return 1;
    } else if (type == IST_MaxEquivalentStrainLevel) {
        answer.resize(1);
        answer.at(1) = status->giveKappa ();
        return 1;
    } else return StructuralMaterial::giveIPValue (answer, aGaussPoint, type, atTime);
}

InternalStateValueType
IsotropicDamageMaterial::giveIPValueType (InternalStateType type)
{
    if ((type == IST_DamageTensor)) return ISVT_TENSOR;
    else return StructuralMaterial::giveIPValueType (type);
}

int
IsotropicDamageMaterial::giveIntVarCompFullIndx (IntArray& answer,

```

```

                                InternalStateType type,
                                MaterialMode mmode)
{
    if ((type == IST_DamageTensor)) {
        answer.resize (9);
        answer.at(1) = 1;
        return 1;
    } else
        return StructuralMaterial::giveIntVarCompFullIndx (answer, type, mmode);
}

int
IsotropicDamageMaterial::giveIPValueSize (InternalStateType type,
                                           GaussPoint* aGaussPoint)
{
    if ((type == IST_DamageTensor)) return 1;
    else return StructuralMaterial::giveIPValueSize (type, aGaussPoint);
}

void
IsotropicDamageMaterial :: giveThermalDilatationVector (FloatArray& answer,
                                                         GaussPoint * gp,
                                                         TimeStep* tStep)
//
// returns a FloatArray(6) of initial strain vector
// eps_0 = {exx_0, eyy_0, ezz_0, gyz_0, gxz_0, gxy_0}^T
// caused by unit temperature in direction of
// gp (element) local axes
//
{
    answer.resize (6); answer.zero();
    answer.at(1) = this->tempDillatCoeff;
    answer.at(2) = this->tempDillatCoeff;
    answer.at(3) = this->tempDillatCoeff;

    return ;
}

IRResultType
IsotropicDamageMaterial :: initializeFrom (InputRecord* ir)
{
    const char *__proc = "initializeFrom"; // Required by IR_GIVE_FIELD macro
    IRResultType result;                  // Required by IR_GIVE_FIELD macro

    IR_GIVE_FIELD (ir, tempDillatCoeff, IFT_IsotropicDamageMaterial_talpha, "talpha"); // Macro
    return StructuralMaterial::initializeFrom (ir);
}

```

In the last part, the implementation of the **IsotropicDamageMaterialStatus** class services follows. The *init-TempStatus* and *updateYourself* services are used to initialize temporary history variables according to the previous

equilibrium state or to update the non-temporary variables, when a new equilibrium is reached. Also, the implementation of *storeContext* and *restoreContext* services is shown. The service *printYourself* is also overloaded. It allows to print history variables into output file. Note that all overloaded methods call the corresponding method of the base class (to ensure that processing corresponding to the base class is performed, to guarantee consistency) and then the required functionality is added.

```

IsotropicDamageMaterialStatus::IsotropicDamageMaterialStatus (int n,
                                                                Domain* d,
                                                                GaussPoint* g)
:StructuralMaterialStatus(n,d,g)
{
    kappa = tempKappa = 0.0;
    damage = tempDamage = 0.0;
}

IsotropicDamageMaterialStatus::~IsotropicDamageMaterialStatus ()
{}

void
IsotropicDamageMaterialStatus :: printOutputAt (FILE *file, TimeStep* tStep)
{
    StructuralMaterialStatus :: printOutputAt (file, tStep);
    fprintf (file,"status { ");
    if (this->damage > 0.0) {
        fprintf (file,"kappa %f, damage %f ",this->kappa, this->damage);
    }

    fprintf (file,"}\n");
}

void
IsotropicDamageMaterialStatus::initTempStatus ()
{
    StructuralMaterialStatus :: initTempStatus();
    this->tempKappa = this->kappa;
    this->tempDamage= this->damage;
}

void
IsotropicDamageMaterialStatus::updateYourself(TimeStep* atTime)
{
    StructuralMaterialStatus::updateYourself(atTime);
    this->kappa = this->tempKappa;
    this->damage= this->tempDamage;
}

contextIOResultType

```

```

IsotropicDamageMaterialStatus::saveContext (FILE* stream, void *obj)
{
    contextIOResultType iores;

    // save parent class status
    if ((iores = StructuralMaterialStatus :: saveContext (stream, obj))
        != CIO_OK) THROW_CIOERR(iores);

    // write a raw data
    if (fwrite(&kappa,sizeof(double),1,stream) != 1) THROW_CIOERR(CIO_IOERR);
    if (fwrite(&damage,sizeof(double),1,stream)!= 1) THROW_CIOERR(CIO_IOERR);

    return CIO_OK;
}

contextIOResultType
IsotropicDamageMaterialStatus::restoreContext(FILE* stream, void *obj)
{
    contextIOResultType iores;

    // read parent class status
    if ((iores = StructuralMaterialStatus :: restoreContext(stream,obj))
        != CIO_OK) THROW_CIOERR(iores);

    // read raw data
    if (fread (&kappa,sizeof(double),1,stream) != 1) THROW_CIOERR(CIO_IOERR);
    if (fread (&damage,sizeof(double),1,stream) != 1) THROW_CIOERR(CIO_IOERR);

    return CIO_OK;
}

```

The above general class provides the general framework. The derived classes have to implement only services *computeEquivalentStrain* and *computeDamageParam*, which are only related to particular material model under consideration.

5 Dof Managers, DOFs, Boundary and Initial Conditions

5.1 DOFs

Abstract class **Dof** represents Degree Of Freedom in finite element mesh. DOFs are possessed by DofManagers (i.e, nodes, element sides or whatever) and one **Dof** instance belongs to only one **DofManager** instance. Dof maintain its physical meaning and reference to related DofManager (reference to DofManager which possess particular DOF). To describe physical meaning of particular Dof, special enum type "DofId" has been introduced (see cltypes.h). This type is more descriptive than UnknownType, which determines physical meaning for unknowns generally (displacement or temperature). DofId type has to distinguish between DOFs representing displacement, but in different directions, since only some of those may be required by particular elements.

Dof can be subjected to boundary (BC) or initial (IC) condition. Method for obtaining corresponding DOF unknown value is provided. If no IC condition has been given, zero value IC is assumed otherwise when needed.

Dof class generally supports changes of static system during computation. This feature generally leads to equation renumbering. Then because equation number associated to dof may change, it may become extremely complicated to ask EngngModel for unknown from previous time step (because equation number may have been changed). To overcome this problem, derived class will implement so called unknown dictionary, which is updated after finishing each

time step and where unknowns for particular dof are stored. Dof then uses this dictionary for requests for unknowns instead of asking EngngModel for unknowns. Unknowns in dof dictionary are updated by EngngModel automatically (if EngngModel supports changes of static system) after finishing time step.

The base class **Dof** declares many services necessary for DOF handling. Some of them are abstract, they have to be implemented by derived classes representing “specialised” DOF types. This basic DOF interface includes the following services

- Services for requesting corresponding unknowns (*giveUnknown*) and the prescribed boundary values (*giveBcValue*).
- Methods for requesting associated equation number (*giveEquationNumber*).
- Services for requesting DOF properties (*hasBc*, *hasIc*, *giveDofID*, *giveDofIDName*, *giveUnknownType*, *isPrimaryDof*).
- Services for dof output (*printOutputAt*, *printStaticOutputAt*, *printDynamicOutputAt*), update after new equilibrium is reached (*updateYourself*, *updateUnknownsDictionary*) and services for storing/restoring context (*saveContext*, *restoreContext*).

The derived classes should implement specific types of DOFs. The most common types are provided with OOFEMlim. Supported are so-called MasterDofs, representing true DOFs with their own equation number, Slave DOFs, representing single DOF linked to another single DOF, Rigid Arm Dofs allowing to model rigid arms (single dof linked to one or more other DOFs). Their brief description follows:

- **MasterDof** - class representing “master” degree of freedom. Master is degree of freedom, which has its related unknown and corresponding equation number. It maintain its equation number.
- **SlaveDof** - class representing “slave” degree of freedom. Slave dof is linked to some master dof (link slave-slave is not allowed). Slaves can be used to implement duplicated joints (by specifying some dof in node as masters and some as slaves linked to dofs in other node (with same or possibly different coordinates). Slave dof share the same equation number with master. Almost all operation as request for BC or IC or equation number are simply forwarded to master. Other functions (which change internal state) like *updateYourself*, *updateUnknownsDictionary*, *askNewEquationNumber* or context storing/restoring functions are empty functions, relying on fact that same function will be called also for master. From this point of view, one can see slave dof as link to other dof.
- **RigidArmSlaveDof** - class representing “rigid arm slave” degree of freedom. This DOF is linked to some master DOFs by linear combination (link slave-slave is not allowed) using rigid arm. Implemented using nodal transformation. The Rigid Arm Slave dof represent DOF, which is directly related to one or more master DOFs. Therefore the rigid arm slave’s equation number is undefined. Similarly, rigid arm slave cannot have own boundary or initial conditions - these are entirely determined using master boundary or initial conditions. The transformation for DOFs and load is not orthogonal - the inverse transformation can not be constructed by transposition. Because of time consuming inversion, methods can generally compute both transformations for dofs as well as loads. It is important to ensure (on input) that both slave dofManager and master dofManager are using the same local coordinate system. In future releases, this can be checked using *checkConsistency* function, where this check could be performed.

5.2 Dof Managers

The **DofManager** class is the base class for all dof managers. Dof manager is an abstraction for object possessing degrees of freedom (DOFs). Dof managers (respectively derived classes, representing nodes or sides) are usually shared by several elements and are maintained by corresponding domain. The elements keep the logical reference to corresponding dof managers (they store their numbers, not the physical links). The base class declares the variable storing total number of DOFs, the *dofArray* representing the list of DOFs managed by dof manager, and *loadArray*

list for storing the applied loadings references. The **DofManager** declares (and implements some of them) following services

- DOF management methods. The methods for requesting the total number of DOFs (*giveNumberOfDofs*), requesting particular DOFs (*giveDof*), assembling location array of code numbers (*giveLocationArray* and *giveCompleteLocationArray*), methods for DOF selection based on their physical meaning (*giveDofArray* and *findDofWithDofId*), and services for requesting unknowns related to dof manager DOFs (*giveUnknownVector* and *givePrescribedUnknownVector*).
- Transformation functions. The governing equations can be assembled not only in global coordinate system, but also in user-defined local coordinate system of each dof manager. Following methods introduce necessary transformation methods, allowing DOFs to be expressed in their own local c.s. or to be dependent on other dofs on other dofManager (to implement slave or rigid arm nodes etc.). The methods for computing transformation matrices from global c.s to receiver's defined coordinate system are declared (*computeDofTransformation* and *computeLoadTransformation*). The *requiresTransformation* indicates whether dofManager requires the transformation from global c.s. to dof manager specific coordinate system.
- Load management functions. The two methods are provided for handling applied loading - the service for computing the load vector of receiver in given time (*computeLoadVectorAt*) and service providing the list of applied loads (*giveLoadArray*).
- Context related services for storing/restoring the receiver state to context (*saveContext* and *restoreContext* services) are implemented.
- Instantiating service *initializeFrom*.
- Miscellaneous services for receiver printing, identification, etc.

In the OOFEMlib the common specialized dof managers are defined and implemented. Those currently provided are:

- **Node** class. Class implementing node in finite element mesh. Node manages its position in space, and if specified, its local coordinate system in node. If local coordinate system is defined, all equilibrium equations are assembled in this system and therefore all DOFs and applied boundary and initial conditions apply in this local coordinate system. By default, global coordinate system is assumed in each node.
- **RigidArmNode** class. Class implementing node connected to other node (master) using rigid arm in finite element mesh. Rigid arm node supports not only slave dofs mapped to master but also some dofs can be primary dofs. The masterDofMask attribute is introduced allowing to distinguish between primary and mapped (slave) dofs. The primary DOFs can have their own boundary and initial conditions. The introduction of rigid arm connected nodes allows to avoid very stiff elements used for modelling the rigid-arm connection. The rigid arm node maps its dofs to master dofs using simple transformations (small rotations are assumed). Therefore, the contribution to rigid arm node are localized directly to master related equations. The rigid arm node slave (mapped dofs can not have its own boundary or initial conditions, they are determined completely from master dof conditions. The local coordinate system in slave is not supported in current implementation, the global lcs applies. On the other hand, rigid arm node can be loaded independently of master. The transformation for DOFs and load is not orthogonal - the inverse transformation can not be constructed by transposition. Because of time consuming inversion, methods can generally compute both transformations for dofs as well as loads.
- **ElementSide** class - representing finite element side possessing some DOFs.

5.3 Boundary Conditions

The library introduces the base abstract class **GeneralBoundaryCondition** for all boundary conditions (both primary and secondary). Boundary condition is an attribute of the domain (it belongs to). The other system components subjected to boundary conditions keep reference to corresponding boundary conditions, like elements, nodes, and Dofs. The base class only declares itself as a base class of all boundary conditions, and declares only very basic services. It introduces 'loadTimeFunction' as an attribute of each boundary condition. 'loadTimeFunction' represent time variation, its value is dependent on time step. The value (or the components) of a boundary condition (load) will be the product of its value by the value of the associated load time function at given time step. The meaning of boundary condition components is dependent on particular boundary condition type, and should be defined in derived classes documentation. This base class introduces also two general services for requesting boundary condition physical meaning and boundary condition geometrical character (pointwise, acting on element body or edge and so on). Derived classes should represent the base classes for particular boundary condition type (like force load, or boundary condition prescribed directly on some dof) and should declare the basic common interface. For example, the **Load** is derived from **GeneralBoundaryCondition** and represent base class for all load types. The following derived classes are provided by OOFEMlib

- **Load** class - base abstract class for all loads. Declares the attribute componentArray to store load components and method for evaluating the component values array at given time (component array multiplied with load time function value) is provided.
- **NodalLoad** - implementation of a concentrated load (force, moment,...) that acts directly on a dof manager (node or element side, if it has associated DOFs). This load could not be applied on an element. A nodal load is usually attribute of one or more nodes or element sides.
- **BoundaryLoad** - abstract base class representing a boundary load (force, momentum, ...) that acts directly on a boundary of some finite element (on element side, face, ...). Boundary load is usually attribute of one or more finite elements. This base class only declares the common services to all derived classes. Derived classes must implement abstract services and possibly may customize existing. Boundary load is represented by its geometry (determined by its type - linear, quadratic load) and values (it is assumed, that user will supply all necessary values for each dof).

The load can generally be specified in global space or can be related to local entity space (related to edge, surface). If load is specified in global space then its values are evaluated at points, which is characterized by global coordinates. If load is specified in entity space, then point is characterized by entity isoparametric coordinates.

Methods for evaluation of load component values in any point on element boundary (on side, face, ...) are provided (this point is determined using global coordinates). The other possibility (faster, but less general) is to specify the point using isoparametric coordinates - but this is not supported. It is generally assumed, that derived classes will approximate somehow their values based on user specified data (on side nodes for example) and load approximation type. The similar scheme borrowed from FE approximation is used, values computed at required point are computed as a product of approximation matrix (matrix of approximation functions) with "vertex" values, which has to be specified on input by user. Elements can request the order of load approximation (for setting up the appropriate integration rule order) and the array of component values (for each dof) at specific integration point on the boundary.

For some elements it may be better to obtain "vertex values" of boundary load to compute load vector directly using exact formulae. Elements then can ask for values at nodal points and obtain corresponding "vertex values". Meaning of these values is load type dependent, see derived classes documentation for details.

Elements must take care, on which boundary the load acts on (side number, ...). Boundary load class also introduces load related coordinate system indicator. Load can be generally specified in global coordinate

system or in entity dependent local coordinate system. The entity dependent coordinate system is defined by particular element.

To summarize, the services provided include computing component array evaluated at specific point on boundary, returning component array of "vertex values", returning load approximation order (useful when numerical integrations of load vector over element boundaries are used), returning type of coordinate system, in which load applies (global c.s., or entity related c.s.).

- **BodyLoad** - Class representing base class for all element body load, acting over whole element volume (e.g., the dead weight).
- **BoundaryCondition** - class implementing boundary condition on DOF (primary boundary condition). This boundary condition is usually attribute of one or more degrees of freedom (DOF). The type of unknown (physical meaning) is fully determined by corresponding DOF, to which given BC is associated. Boundary condition can change its value in time using its inherited `loadTimeFunction` attribute. It can also switch itself on or off depending on nonzero value of introduced `isImposedTimeFunction` load time function. Please note, that previous option must be supported by particular engineering model (because equation renumbering is necessary, and for incremental solution schemes DOFs unknown dictionaries must be used). See particular engineering model documentation for details.
- **InitialCondition** - class implementing general initial condition. Initial condition is usually attribute of one or more degrees of freedom (DOFs). One particular DOF (with its physical meaning - for example displacement) can have associated only single initial condition. Initial condition therefore must represent several initial conditions for particular DOF (for example velocity and acceleration of unknown can be prescribed using single initial condition instance). These multiple entries are distinguished by their `CharTypeMode` value. The `CharTypeMode` value is also used as key in `initialValueDictionary`. Initial conditions apply and should be taken into account only in one particular time step, which number is determined from engineering model *giveNumberOfTimeStepWhenIcApply* service. If in this time step both boundary condition on unknown and also initial condition on value of this unknown (`TotalMode CharTypeMode`) are prescribed, then always value reported by boundary condition is taken into account.

6 XFEM module

Design of object-oriented application consists in finding classes and objects, identifying structures and attributes, and defining required services. From the description of XFEM principles, given in the previous section, one can identify the basic classes that represent the fundamental building blocks.

The model entity, described or represented by one or more enrichment functions is represented by hierarchy of classes derived from base **EnrichmentItem** class. Such entity may represent crack, material interface, slipping interface, etc. Each **EnrichmentItem** keeps a list of its enrichment functions. The geometry (spatial location) of individual enrichment items can be described in number of ways. This includes fixed explicit descriptions (circular inclusion, fixed slip line, etc), or general representations based on polygonal, spline, or level set descriptions. To achieve flexibility of geometry representation of individual **EnrichmentItems**, that allows to choose the optimum for particular purpose, the hierarchy of classes derived from base **Geometry** class is designed. The abstract interface, declared by parent **Geometry** class defines general methods to compute receiver signed distance from given point, methods to test whether receiver intersects given element, methods to compute intersection points, etc. The instance of **Geometry** class is an attribute of **EnrichmentItem** class.

The global shape functions are represented by classes derived from parent **EnrichmentFunction** class. The parent class declares abstract interface common to all **EnrichmentFunction** implementations. Particularly, it defines *evaluateAt* and *evaluateDerivativeAt* methods to evaluate the value and spatial derivatives at a given point. Moreover, it also defines a method *resolveLinearDependency* at a node. Linear dependency typically occurs, when a nodal support

is cut into two disjoint pieces and the area of one piece is very small compared to the other piece[?]. This is usually resolved by excluding the node from the enrichment.

At the level of problem representation, the **XfemManager** has been introduced. It manages the list of active enrichment items. Its interface provides initialization and access services to individual enrichment items. When a certain node in the problem domain is subjected to enrichment, the corresponding enrichment item has to introduce additional degree(s) of freedom in that node. Since several enrichment items can be active in a node, all DOFs are assigned with a label, that allows to distinguish between them (in traditional FE codes labels are also used to distinguish physical meaning of particular DOF representing displacement along x,y,z axes, rotations, etc.). The **XfemManager** class maintains a pool of available labels and is responsible for their assignment to individual enrichment items. As many enrichment items evolve in time, depending on the solution state, the method to update their state and geometry at the end of each solution step is provided (by calling corresponding update method of each **EnrichmentItem**). Also, other useful services such as tests whether node/element is enriched with given enrichment item are provided. The basic reason for introducing **XfemManager** class is to wrap XFEM related matter at the problem level within a single class. This avoids to a certain extent the need to modify and penetrate the original code, keeping its structure and interface simple. The graphical representation of mutual relations between aforementioned classes is shown in Fig. 5 using UML notation (see [?] for reference).

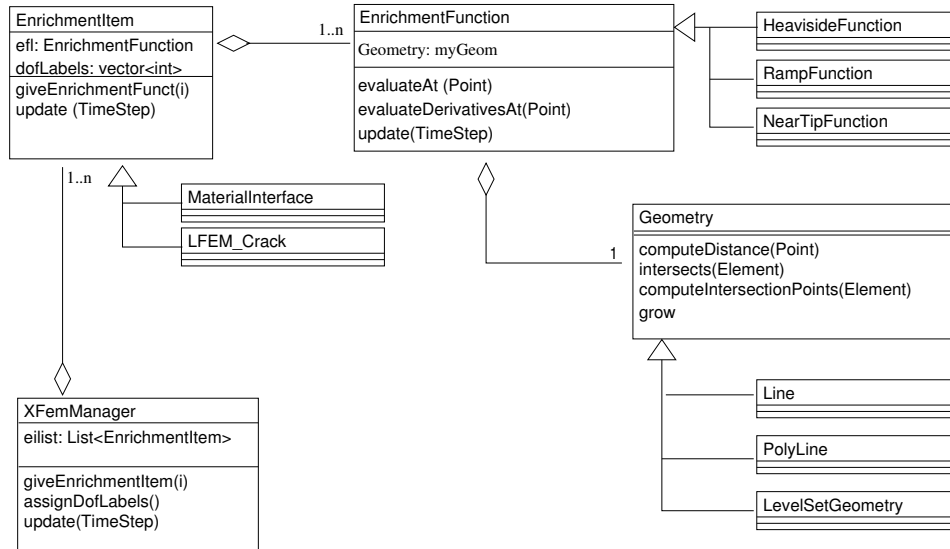


Figure 5: Collaboration diagram for **EnrichmentItem** class

Each enrichment item keeps its assigned DOF labels (provided by XfemManager). These are used by individual enrichment items to introduce and later identify related DOFs in individual nodes. The physical meaning of these DOFs depends on the enrichment item. For example, a single DOF is needed to represent slip along slip line, two DOFs are required for displacement discontinuity in 2D (normal and tangential displacement jumps), etc. Each enrichment item has to implement its *update* method. This method implements all the physics behind, driving for example the crack growth or interface evolution. It is responsible for updating geometry, enriching corresponding nodes, and by collaboration with XfemManager to trigger the update of integration schemes at enriched elements. .

A degree of freedom in the standard FEM implementation is represented by a class **DOF**. Each **DOF** maintains its label, reference to the node it belongs to, list of applied boundary and initial conditions and its equation number. When the particular enrichment item evolves, additional elements and corresponding nodes are subjected to enrichment. At node level, additional DOFs corresponding to enrichment item need to be introduced. This is the responsibility of

EnrichmentItem::update method. In collaboration with **XfemManager**, the new **DOF** instance is created, assigned with proper label and equation number (next available equation number obtained from problem representation) and added into corresponding node. The introduction of new DOFs during solution also leads to increase of number of equations. This requires some support from underlying FEM system.

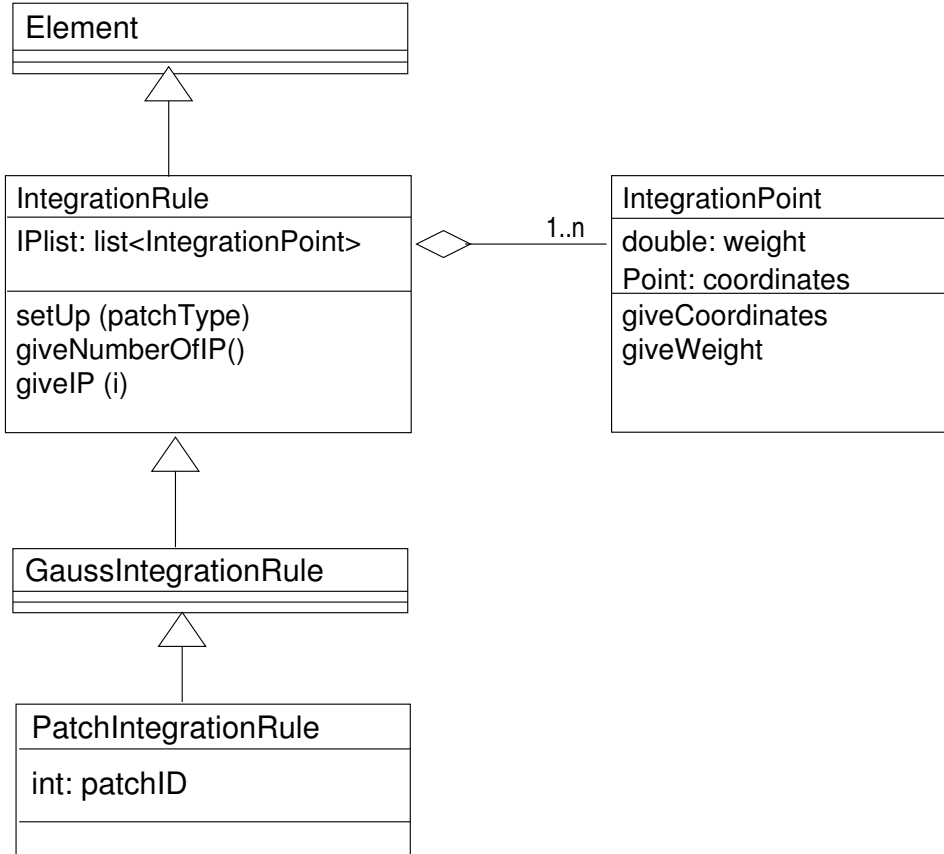


Figure 6: Collaboration diagram for **IntegrationRule** class

Standard FEM implementation at element level has to be extended in order to manage additional complexity introduced by extended formulation. This includes handling of additional degrees of freedom due to enrichment, integration, treating additional terms in element characteristic matrices, etc. Most of the XFEM complexity at element level is encapsulated into **XfemElementInterface** class. The elements with XFEM functionality are derived from standard **Element** base class, as well as from **XfemElementInterface**. In this way the design strives for the least possible interception with the current FEM code.

In order to provide a high-level abstraction for different integration techniques and to make the element implementation independent of particular integration technique and transparent as much as possible, the **IntegrationRule** class is proposed. This class maintains its integration points, which are represented by class **IntegrationPoint**. Integration points maintain their spatial coordinates and corresponding integration weights. Integration rule is responsible for setting up the individual integration points and for providing access methods to these points.

An important feature of element implementation is the possibility to set up and maintain several integration rules. In standard FE implementation this is used to implement reduced or selective integration. This concept has been

found useful also in present context. For example, the commonly used sub-patch integration scheme is represented by sequence of instances of **PatchIntegrationRule** class. As Gauss integration is typically employed on individual patches, the **PatchIntegrationRule** can be conveniently derived from existing **GaussIntegrationRule** class (see Fig. 6). When initializing the integration points, their local coordinates (related to patch geometry) are mapped into parent element coordinates and corresponding integration weights are scaled accordingly. There is no need to store individual patch geometries, as the parent element geometry is used to correctly evaluate transformation Jacobians. This concept also allows to easily accommodate additional integration points, if needed. For example, in the case of cohesive crack model, the additional integration points along the discontinuity are needed. Then the additional integration rules (representing discontinuity line) can be introduced into the list of element integration rules. The example of stiffness matrix evaluation at element level, which can be used both by classical and enriched elements and profits from aforementioned concept, is presented in Tab. 1 using symbolic code.

FloatMatrix* PlaneStress2d::computeStiffnessMatrix()

```

loop on all IntegrationRules of the Element:
  loop on all Gauss points of the IntegrationRule:
    B = this->computeStrainDisplMatrix(gp)
    D = this->computeConstitutiveMatrix(gp)
    dV = computeVolumeAround(gp)
    stiffnessMatrix->add( $B^T D B$  dV)

```

Table 1: Stiffness matrix evaluation

The implementation of stiffness matrix evaluation assumes, that the strain - displacement matrix contains all the terms, i.e. the standard terms as well as terms due to the enrichment. The implementation of method for computing plane stress strain displacement matrix is presented in the Tab. 2. The matrix comes with more columns to account for additional DOFs due to enrichment. The stiffness matrix evaluation, implemented using classes representing real valued matrices and corresponding methods, can therefore work with any matrix size, provided that the sizes of individual matrices are compatible.

FloatMatrix* PlaneStress2d::computeStrainDisplMatrix(GaussPoint* gp)

```

compute standard part of the strain-displ. matrix at gp

loop on EnrichmentItems of the element nodes:
  loop on element nodes:
    if node is enriched by enrichment item:
      give enrichment function(s) of the enrichment item
      loop on the number of dofs of an enrichment function(s):
        add the contribution of the enrichment function
        to the strain-displacement matrix

```

Table 2: Strain-displacement matrix evaluation

7 Design of IGA Module

In the traditional FEA context, **Element** class is parent class for finite elements. It maintains list of nodes, boundary conditions (loads), integration rules, it keeps links to its interpolation and associated material model and provides general services for accessing its attributes, methods for returning its code numbers, and abstract services for the evaluation of characteristic components (e.g. stiffness matrix, load vector, etc.). In the proposed IGA module, the individual NURBS (or B-spline) patches are represented by classes derived from the base **IGAElement** class which is in turn derived from **Element** class. As a consequence, the B-spline patch is represented and treated as a single entity, which returns its contributions that are localized into global system of characteristic equations. This is important also in the sense of slightly different handling of primary unknowns (see Section ??), compared to the standard FEA. Moreover, it is quite natural because in a general case, degrees of freedom (DOFs) on all nodes (control points) of the patch may interact with each other.

One of the fundamental issues, that has to be addressed at the element level, is the integration. The integration, usually the Gaussian integration, is performed over the nonzero knot spans. Individual integration points are represented by **GaussPoint** class. They maintain their spatial coordinates and corresponding integration weights. The individual integration points are set up and grouped together by base **IntegrationRule** class. For example, **GaussIntegrationRule** class, derived from **IntegrationRule** base, implements Gauss quadrature. An important feature is that an element may have several integration rules. This is very useful for the implementation of the reduced or selective integration, for example. The concept of multiple integration rules per element is extended in the present context of the IGA. The **IGA_IntegrationElement** class is introduced to represent the integration rule for individual nonzero knot spans, see Figure 7. Since it is derived from **GaussIntegrationRule**, it inherits the capability to set up Gauss integration points. The reason for creating a new class is to introduce the new attribute `knotSpan`, where the corresponding knot span is stored. This information is used in interpolation class to evaluate corresponding basis functions and their derivatives at a given integration point. Note that generally, the active knot span can be determined for each integration point on the fly whenever it is needed, but in our approach this information is stored to save the computational time. The **IGA_IntegrationElement** instances are set up for each nonzero knot span. The element integration then consists of a loop over individual nonzero knot spans, i.e. the loop over **IGA_IntegrationElements** and by inner loop over the individual integration points of the particular knot span.

The individual IGA elements are derived from **IGAElement** class, derived from the base **Element**. The purpose of **IGAElement** class is to provide general method for initialization of individual integration rules on nonzero knot spans (represented by **IGA_IntegrationElement** class). The integration rules provide methods to create individual integration points in parametric space. An efficient implementation requires to map coordinates of individual integration points with parametric coordinates related to knot spans directly to knot vector based space. Specific elements are derived from the base **Element** (or **IGAElement** class), that delivers the generic element part and also from one or more classes implementing problem-specific functionality, see Figure 8. In the presented approach, **StructuralElementEvaluator** is an abstract base class, that defines the interface for structural analysis, which includes

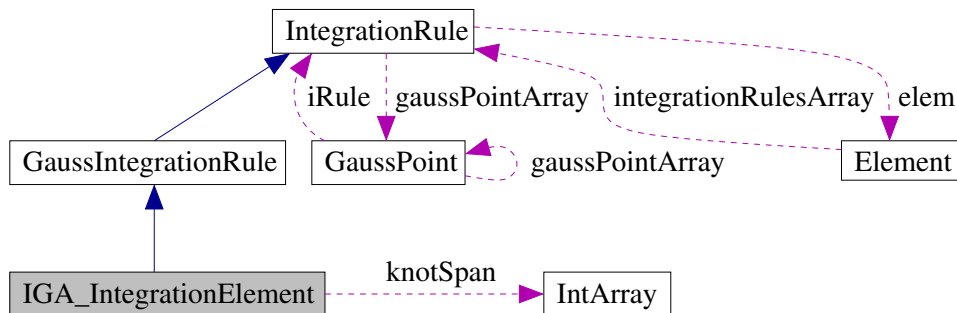


Figure 7: Collaboration diagram for **IGA_IntegrationElement** class.


```

StructuralElementEvaluator::computeStiffnessMatrix(){
  loop on all integration rules of the element:
    loop on all Gauss points of the IntegrationRule:
      B = this->computeStrainDisplacementMatrix(gp);
      D = this->computeConstitutiveMatrix(gp);
      dV = this->computeVolumeAround(gp);
      stiffnessMatrix->add(product of B^T_D_B_dV);
}

```

Table 3: Symbolic code for the evaluation of the stiffness matrix (keyword “this” means that the called method is provided by the class itself).

methods to evaluate mass and stiffness matrices, load vectors, etc. Some of the methods are already implemented at this level, such as stiffness matrix evaluation, based on declared abstract services (evaluation of strain-displacement matrix, etc.), which have to be implemented by derived classes. The example of the evaluation of the element stiffness matrix, which can be used by both classical and IGA based elements, is presented in Table 3 using symbolic code.

In the structural analysis context, classes derived from **StructuralElementEvaluator** implement desired functionality for specific types of structural analyzes (plane-stress, plane-strain, full 3D, etc). Provided that the element defines its interpolation (B-spline basis functions), it is possible to evaluate remaining abstract methods from **StructuralElementEvaluator** interface. This is illustrated in Table 4 on an example of the evaluation of the strain-displacement matrix for the case of plane stress analysis. Thus, when a new element is defined, it has to create its own interpolation and should be derived from **Element** class, which delivers the general basic element functionality and from one or more evaluators, implementing analysis-specific functionality. Such a design, based on decoupled representation of element interpolation and problem specific evaluators, has several advantages. It allows to define problem specific methods only once for several elements with different geometry and interpolation and allows straightforward implementation of elements for multi-physics simulations.

The description of the element interpolation is encapsulated into a class derived from **FEInterpolation** class which defines the abstract interface in terms of services that evaluate shape functions, their derivatives, jacobian matrix, etc. at given point. In the frame of presented work, the **BSplineInterpolation** class has been implemented. Each finite element has to set up its interpolation and provide access to it. This is enforced by general **Element** interface, that requires to define the method for accessing element interpolation. The abstract **FEInterpolation** class interface is essential, as it allows to implement problem specific element methods already at the top level (like the evaluation of element interpolation or strain-displacement matrices). An efficient implementation should profit from the locality of individual interpolation functions which have limited support over several consecutive knot spans. Therefore methods declared by **FEInterpolation** class evaluating values of interpolation functions or their derivatives return the values only for those, that are nonzero in actual knot span. This enables to compute characteristic element contributions on a knot span basis efficiently. For each individual knot span, the contributions are computed only for generally nonzero shape functions and then are localized into element contribution. The mask of nonzero shape functions for individual

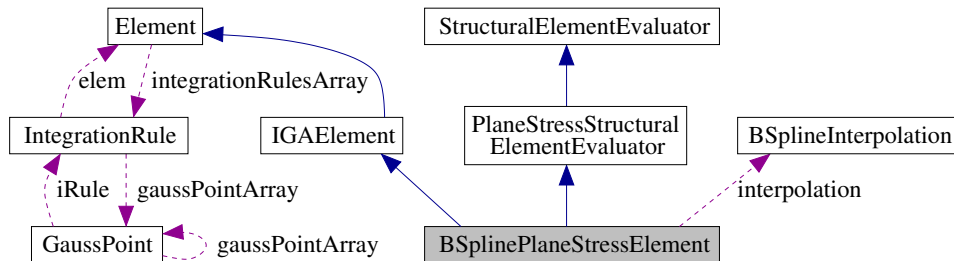


Figure 8: Collaboration diagram for **BSplinePlaneStressElement** class.

```

PlaneStressStructuralElementEvaluator::
computeStrainDisplacementMatrix(IntegrationPoint gp) {
    FEInterpolation interp = gp->giveElement()->giveInterpolation();
    interp->evalShapeFunctDerivatives (der, gp);

    answer.resize(3, nnode*2);          // 2 DOFs per each node
    answer.zero();

    for i=1:nnode
        // epsilon_x
        answer.at(1, i*2-1) = der.at(i, 1);    // dN(i)/dx
        // epsilon_y
        answer.at(2, i*2)   = der.at(i, 2);    // dN(i)/dy
        // shear strain
        answer.at(3, i*2-1) = der.at(i, 2);    // dN(i)/dy
        answer.at(3, i*2)   = der.at(i, 1);    // dN(i)/dx
    }

```

Table 4: Symbolic code for the evaluation of the strain-displacement matrix.

knot spans can be evaluated using *giveKnotBasisFuncMask* service declared by **FEInterpolation** and really provided by **BSplineInterpolation**.

8 Miscellaneous

8.1 Vectors and Matrices

The OOFEMlib provides the abstraction for integer vectors (**IntArray**) and for real vectors and matrices (**FloatArray** and **FloatMatrix**). The usual arithmetic operations (like vector and matrix additions, subtractions, matrix vector multiplication, matrix inverse, solution of linear system, finding eigenvalues and eigenvectors) are provided. However, the usual math operators ('+', '*') are not overloaded, user has to call specific routines. The vector and matrices provide both 0-based and 1-based component access, they allow for dynamical resize with optimal chunk. In fact, the current implementation of *resize* only grows the receiver, the possible request for shrink does not cause the reallocation of memory, since allocated memory is kept for future possible resize. If resize operation wants to force allocation, then *hardResize* should be invoked instead. The preferred argument passing is by reference, even for function or procedure return values. The called function performs resize and fills up the return parametr(s). This is motivated by aim to avoid memory allocation/deallocation problems. The programmer should always avoid to return pointers to newly allocated arrays or matrices. See section “Coding Standards” for details.

8.2 Solution Step

Class representing solution step. The Solution step instance may represent either time step, load increment, or load case depending on current Engineering model used.

Solution step maintain the reference to corresponding Engineering model class instance. It maintain also its “intrinsic time” and corresponding time increment. The meaning of these values is dependent on current Engineering model used. The time may represent either current time, load increment number or load case number. See corresponding Engng model reference for details.

Some components (typically integration points real stresses or integration points nonlocal values) are computationally very demanding. Because in typical code, there are number of requests for same value during the computation process, it may be efficient to store these values and compute them only once. The principal problem is to recognize, when is necessary to re-compute these stored values to reflect newly reached state. This cannot be determined from solution step “time”, because solution step may represent for example load increment, inside which typically many iterations are

needed to reach convergence. For this purpose, a concept of solution state counters is provided. Whenever the solution state changes, the engineering model updates the solution state counter. The solution state counter is guaranteed to grow up smoothly (it never decreases) during solution process. Other components of program (integration points) can then store their computationally expensive values but have to store also corresponding solution state counter value valid when these were computed. Then they can easily check for difference between frozen solution state counter for their value with current solution state requested from solution step and recompute the values if necessary.

8.3 Load Time Functions

Abstract base class representing load time function. Classes derived from Load class typically describe load from spatial point of view. The purpose of introducing load time function is to express variation of some components in time. Load time function typically belongs to domain and is attribute of one or more loads. Generally load time function is real function of time ($y = f(t)$).

9 Coding Standards

9.1 Naming Conventions

The names of classes, attributes, services, variables, and functions in a program serve as comments of a sort. So don't choose terse names—instead, look for names that give useful information about the meaning of the variable or function. In a OOFEM program, names should be English, like other comments.

Local variable names can be shorter, because they are used only within one context, where (presumably) comments explain their purpose.

Try to limit your use of abbreviations in symbol names. It is ok to make a few abbreviations, explain what they mean, and then use them frequently, but don't use lots of obscure abbreviations.

Please use capital letters to separate words in a name. Stick to lower case; reserve upper case for macros, and for name-prefixes that follow a uniform convention. The function or service name should always begin with lowercase letter, the first uppercase letter in function name indicates, that function is returning newly allocated pointer, which has to be deallocated. For example, you should use names like `ignoreSpaceChangeFlag`;

When you want to define names with constant integer values, use enum rather than '#define'. GDB knows about enumeration constants.

Use descriptive file names. The class declarations should be placed in *.h files and class implementation in corresponding *.C files. For each class, create a separate files.

9.2 Parameters and Return Values

The preferred argument passing method for objects is by reference. Try to avoid returning pointers to arrays or matrices (or generally to any component), since it is not clear, whether to deallocate the returned pointer or not. The most preferred way is to create local variable of vector or matrix type, pass it (using reference) to called function. The calling function is responsible to properly resize the (output) parameter and set values accordingly. The point is, that destructors are called for local variables automatically by compiler, so there is no possibility for memory leaks and the local variable can be reused for multiple calls to target function (inside loop) and therefore there is no need for repeating memory allocation and deallocation. Sometimes it may be reasonable to return pointer to constant float array or matrix (representing for example nodal coordinates), since passing output array as parameter will require array copying.

9.3 Interface concept

Interfaces provide a way, how to organize or group optional or function-specific services into clearly defined units (well structured code) which can be selectively implemented by some classes.

The interface concept can be illustrated on following simple example. Let's have an error estimator, represented by corresponding class. This estimator requires some special services at the element level. Because typically not all elements provide support for this error estimator, it would be wasting of space to declare required services in general element base class. (This will lead to big and extremely general interface declared by base class, with only some services compulsory and with many optional services. This can lead to confusion and very unclear structure). Suggested remedy is to declare corresponding interface to error estimator. The interface should declare all services needed by error estimator. Particular elements, which would like to provide support for error estimator, simply derive itself from corresponding interface and implement services from interface. If error estimator wants to access its interface, it asks element for pointer to its corresponding interface (getInterface method). Once interface is returned, error estimator can use its services to compute response.

Some difficulty may arise, when implementing general interface service, requiring general services from class implementing interface. Then simple "trick" can be made: The interface should declare service giveClass(), which returns the pointer to class which implements interface. After having this pointer, the interface general service can use all (but only public) services of class implementing interface. For example some service utilizing the integration over element volume can be formulated as general, but when interface is defined, there is no connection to element implementing it. This connection can be established by declaring virtual giveElement() service, which returns the pointer to element implementing the interface. Then general implementation at interface class level can be made, using base Element class public services. Then all element classes implementing the interface should overload only giveElement service, but not the whole service with integration.

To facilitate the interface concept, the abstract class **Interface** is introduced. Its role is to provide a base class for all interfaces. The interface represent some well defined additional or optional ability or support, which is provided by some class. The services and attributes required are encapsulated in particular derived interface class, which is derived from base **Interface** class. The class which wants to implement interface will simply inherits corresponding interface class. Each base class, whose derived classes are assumed to implement some interfaces, will declare the virtual getInterface(InterfaceType) method. This method must be implemented by each derived class implementing some interface and returns pointer to corresponding interface. This service then provides general way, how to access particular interface of given class.

Acknowledgments

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